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**THE USE OF THE LANGMUIR PROBE
TO DETERMINE ELECTRON DENSITIES
SURROUNDING RE-ENTRY VEHICLES**



Prepared for:

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FINAL REPORT
FOR A STUDY PROGRAM
TO DETERMINE MAGNETIC PROPERTIES
OF JPL SPACECRAFT ELECTRONIC COMPONENTS

1-61042-1

January 1964

Prepared for
JET PROPULSION LABORATORY,
California Institute of Technology, Pasadena, California
Sponsored by the National Aeronautics and Space Administration
Under NASA Contract NAS 7-100

TEXAS INSTRUMENTS INCORPORATED
Apparatus Division
6000 Lemmon Avenue
P. O. Box 6015
Dallas 22, Texas

10/67

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I. INTRODUCTION

A. History and Purpose of Study Program

Texas Instruments has been engaged in building magnetometers for antisubmarine warfare for a number of years. In connection with recent magnetometer developments, a great deal of effort was directed toward evaluating the magnetic characteristics of components and hardware and also developing new nonmagnetic, or "magnetically clean," devices. This experience was applicable to building "magnetically clean" spacecraft; therefore, our experience and aid was offered to the Jet Propulsion Laboratory in the form of a study program.

B. Work Statement

The result of the offer was Contract No. BE4-217626 for a 4-month study program beginning September 1963. The work statement for this program consisted of:

1. Preparation of a plan for a program to determine the magnetic properties of spacecraft electronics components to include:
 - a. Determination of a list of components to be evaluated based primarily on the JPL Preferred Parts List.
 - b. Determination of a standard test for measuring components.
2. Compilation of a handbook of terms, formulas, units, and measuring methods pertaining to magnetic properties.

3. A study of the possibility of defining a statistical model for the magnetic field of a spacecraft which could be used as a tool in:
 - a. Setting subsystem magnetic field specifications.
 - b. Determining probable stability of measured or postulated magnetic fields.

C. Compliance With Work Statement

Item 1, a plan for a measurement program to determine the magnetic fields of components, is outlined in Section II of this report. The list of components for magnetic evaluation (Appendix A) was based on the JPL Preferred Parts List and component usage lists for the Mariner C spacecraft.

Since Item 2, the handbook, will probably be the most widely circulated portion of this report, it has been bound separately as Texas Instruments Report No. 2-61042-2.

Item 3, the discussion of a statistical model for the magnetic field of a spacecraft, is included as Section III of this report. An explanation of the statistical terms used in Section III is provided in Appendix B.

II. PLAN TO MEASURE MAGNETIC PROPERTIES OF COMPONENTS

A. Objectives

The primary objective of this measurement program will be to compile a Preferred Parts List which designers can use when developing hardware for a spacecraft having a minimum magnetic field.

In order to realize this objective, a number of secondary objectives must be realized

1. Standardize methods of gathering data.
2. Determine the total magnetic field of each component type tested.
3. Determine the magnetization stability of the total magnetic field.
4. Based on (2) and (3), determine practical classifications for nonmagnetic devices.
5. Compile a parts list of nonmagnetic components along with necessary manufacturing controls and test data.
6. Make manufacturers aware of the market and requirements for nonmagnetic components, and lay the groundwork for the modification of standard devices to remove magnetic inclusions.

B. Recommended Plan

1. Study the Problem of Magnetization Stability

One of the primary goals of the present study program has been the determination of a standardized test for measuring magnetic fields of components. An additional problem is determining how stable these characteristics are as the environment varies. Ideally, nonmagnetic components could be found for every requirement; it is probable that components with some degree of permanent magnetization will have to be used. Therefore, it is also necessary to determine how much the magnetization will change during system testing and spacecraft launching.

There are too many unknown factors at present to specify a method or procedure for determining magnetization stability of a component. Therefore, additional study to devise and evaluate methods of measuring magnetization stability should be included in the first phase of the measurements program. The study will consider the various measurable magnetic characteristics which are directly or indirectly connected with the magnetization stability of a component. The various environments (vibration level, temperature, magnetizing force, etc.) will also be evaluated in terms of their effect on magnetization stability. Tests designed to acquire data which may be used to predict or specify magnetization stabilities will then be devised. The most difficult part will be to apply data and results taken in controlled environments to the general problem of magnetization stability during spacecraft test and launch. A great deal of trial and error experimenting will be required, but the effectiveness of the stability measurements will not be fully known until a large amount of documented experience is gained with complete spacecraft.

2. Send Questionnaire to Manufacturers and Order Parts

The majority of the measurements program will involve measuring components. In order that the data be useful, enough information about components must be known to assure that additional identical units can be purchased at a later date. Since the magnetic characteristics depend on raw materials, plating processes, and heat treatment processes, this information must be either known or the process specifications must be controlled.

The questionnaire and cover letter, Figures 1 and 2, are examples of how the required information may be obtained. These would accompany the purchase order for the components to be measured. By directing the inquiry and purchase order to the manager of marketing, the manufacturer is made aware of the requirement and impending market for nonmagnetic components. This procedure should result in better manufacturer cooperation and should aid in setting purchase contingency requirements for future component orders.

DATE: _____

1. Manufacturer _____
2. Item Description _____
3. Mfgr. Part Number _____
4. Is there a control drawing number or order number which, if specified on a future purchase order, would control all manufacturing processes and materials? If so, please give the number and revision letter which pertains to parts sent on this order. _____
5. To help us determine problem areas in magnetic components, please check any of the following ferromagnetic elements or alloys used in the above components. If an assembly drawing showing parts and materials is available, please include it with the component shipment.

_____ Iron	_____ Kovar	_____ Dumet
_____ Nickel	_____ Rodar	_____ Copperweld
_____ Cobalt	_____ Inconel (X, R, etc.)	_____ Invar
_____ Steel (incl. stainless)	_____ Monel	_____ Nilvar
_____ Brass (selectively nonmagnetic)	_____ Elinvar	_____ Ferrites

Other _____

6. If you use any of the above materials, can you substitute a nonmagnetic material such as copper (pure or tinned), magnesium, aluminum, beryllium copper, phosphor bronze, pure silver, Alloy 180, ceramics (alumina or beryllia), etc.? _____
7. Are any plating processes used in which any of the metals listed in Paragraph 5 are also plated in the same bath? Yes _____ No _____
If yes, what is the plating material? _____
8. Since the magnetic properties of a component are primarily due to the case and leads, it is possible to magnetically group components of identical construction but different electrical properties. Do you have other products (or product families) which use the identical package and lead materials and processes as the components supplied on this order?
Yes _____ No _____
If yes, please list on the reverse side of this form or on a separate sheet.
9. Remarks _____

Figure 1. Typical Manufacturer Questionnaire

Manager of Marketing
ABC Company
1231 Elm Street
New York, New York

Dear Sir.

Your product is being considered for a preferred parts list. The list will be used for spacecraft which carry an experiment to measure magnetic fields. The components aboard such spacecraft must be nonmagnetic in order not to interfere with the measurements. Therefore, we are evaluating the magnetic characteristics of components and hardware and will compile the test results into a list of preferred parts.

We wish to evaluate the representative samples of your products listed on the attached purchase order. A questionnaire is also attached which will be used to

1. Assist us in developing a list of nonmagnetic components.
2. Determine what parts of an item are magnetic and whether it is worthwhile to attempt to develop new devices with the magnetic parts replaced with substitute nonmagnetic materials.
3. Make certain that, after an item is evaluated, additional items can be purchased at a later date which will be fabricated exactly as the evaluated units.

Spacecraft carrying magnetic probes will represent an appreciable portion of the future unmanned spacecraft market. Therefore, this inquiry has been directed to you so that proper attention will be given it and you will not inadvertently be eliminated from a future market. Any additional information which might aid our evaluation will also be appreciated.

Very truly yours,

Figure 2. Typical Letter to Manufacturer

The components to be ordered for magnetic evaluation are listed in Appendix A. The list contains necessary ordering information to include manufacturer identification and part number, test quantities and, if applicable, the number of different production lots from which the total test quantities are to be sampled; i. e., a notation of 20-4 indicates that a total of 20 samples are to be tested with five samples taken from each of four different production lots. As testing progresses, additional quantities of some items will need to be tested due to large variations in results or in order to better define the statistical distribution of component magnetic field intensity. The quantities specified are arbitrary, as there is not, as present, sufficient component data to justify statistical sampling by procedures such as given in MIL-STD-105D, Sampling Procedures and Tables for Inspection by Attributes.

While it may be desirable eventually to limit the amount of components on a Nonmagnetic Preferred Parts List, it is strongly recommended that such a limitation not be applied to the components selected for testing. It is quite likely that the size of the Preferred Parts List will be limited by the results of component tests.

3. Component Measurement

One of the goals of this contract was to determine a preferred method for measuring the magnetic effects of electronic components. The prime considerations for a preferred method included accuracy, repeatability, simplicity, high sensitivity (in the order of 0.1 gamma), and relatively low cost. It was also considered highly desirable, although not mandatory, that the measurement system operate in a laboratory environment without elaborate and expensive field cancellation mechanisms such as Helmholtz or Farnsworth coil systems or magnetically shielded enclosures. It was felt that these considerations would provide a measurement technique readily adaptable to use by component manufacturers for quality assurance. The "modified spinner" method described below fulfills these requirements.

A number of factors must be standardized before actual component measurements begin. These factors include:

- Component lead length

- Methods for determining measurement axes for classes of components such as cylindrical bodies, axial leads, transistor cans, relay cans, toroids, etc.

- Test distance from the magnetometer probe to the approximate geometric center of the component.

Therefore, time must be allowed to consider and document standards.

a. Test Apparatus

Component magnetic field intensity will be measured with test equipment similar to that shown in Figure 3. Essentially, this apparatus consists of a vector magnetometer, a mechanism to rotate or spin the component at a fixed rate, a narrowband filter, and a recorder.

The electronic component to be tested is placed in a sample holder and is rotated at approximately 5 revolutions per second. A Hewlett-Packard Model 3529A magnetometer probe senses the vector component of the sample's magnetic field which is parallel to the cylindrical axis of the probe. The combination of the probe and a Hewlett-Packard Model 428B Clip-On DC Milliammeter forms a fairly sensitive flux-gate magnetometer. The 428B provides a low-impedance 5-cps signal proportional to the component magnetic field intensity. This signal is applied to a high-impedance two-channel strip chart recorder through a bandpass filter which enhances the signal-to-noise performance by reducing the effects of low-frequency interference (changes in the geomagnetic field and locally produced magnetic fields such as produced by equipment being moved) and high-frequency interference (especially magnetic fields at 60- and 400-cps power frequencies so often found in laboratories). An event marker is used to relate the direction of the sample's magnetic field to the sample's position in the test fixture.

The spinning mechanism consists of a 1500-rpm fan motor and suitable pulleys to give a rotation frequency of approximately 5 cps. It is essential that all parts of the spinner which rotate at 5 cps be nonmagnetic; if they are not, the spinner itself will produce a 5-cps magnetic field. The rotating rod and all supports should therefore be made of phenolic or wood. The rod itself must be at least 6 feet long to avoid stray fields due to the motor and drive mechanism. Both pulleys are die-cast zinc, and the larger pulley should be spoke-type, rather than solid, to reduce eddy current fields. The sample holder is a polystyrene box and cover secured to the phenolic rod with a 6-32 aluminum or nylon screw. The rod is drilled and tapped for a 6-32 beryllium copper helicoil insert. The component sample is held in place in the sample holder with a nonmagnetic foam rubber or plastic (such as undyed virgin polyurethane foam).

The magnetometer probe is normally furnished with a cable 7 feet long, but longer cables are available. A cable 25 feet long is preferable, since this allows the operator to make adjustments at the meter or recorder without his movements affecting the field at the probe. Since the 3529A probe is a vector field sensor, the effect of the geomagnetic field may be reduced by orienting the probe orthogonally to the ambient magnetic field. The sample holder is located so that the geometric center of the test sample is a fixed standard distance from the probe tip. If the probe-to-sample distance is at least three times the largest dimension of the sample, the inverse-cube relationship for the field of a dipole will be valid, and the equivalent field may then be computed at any distance.

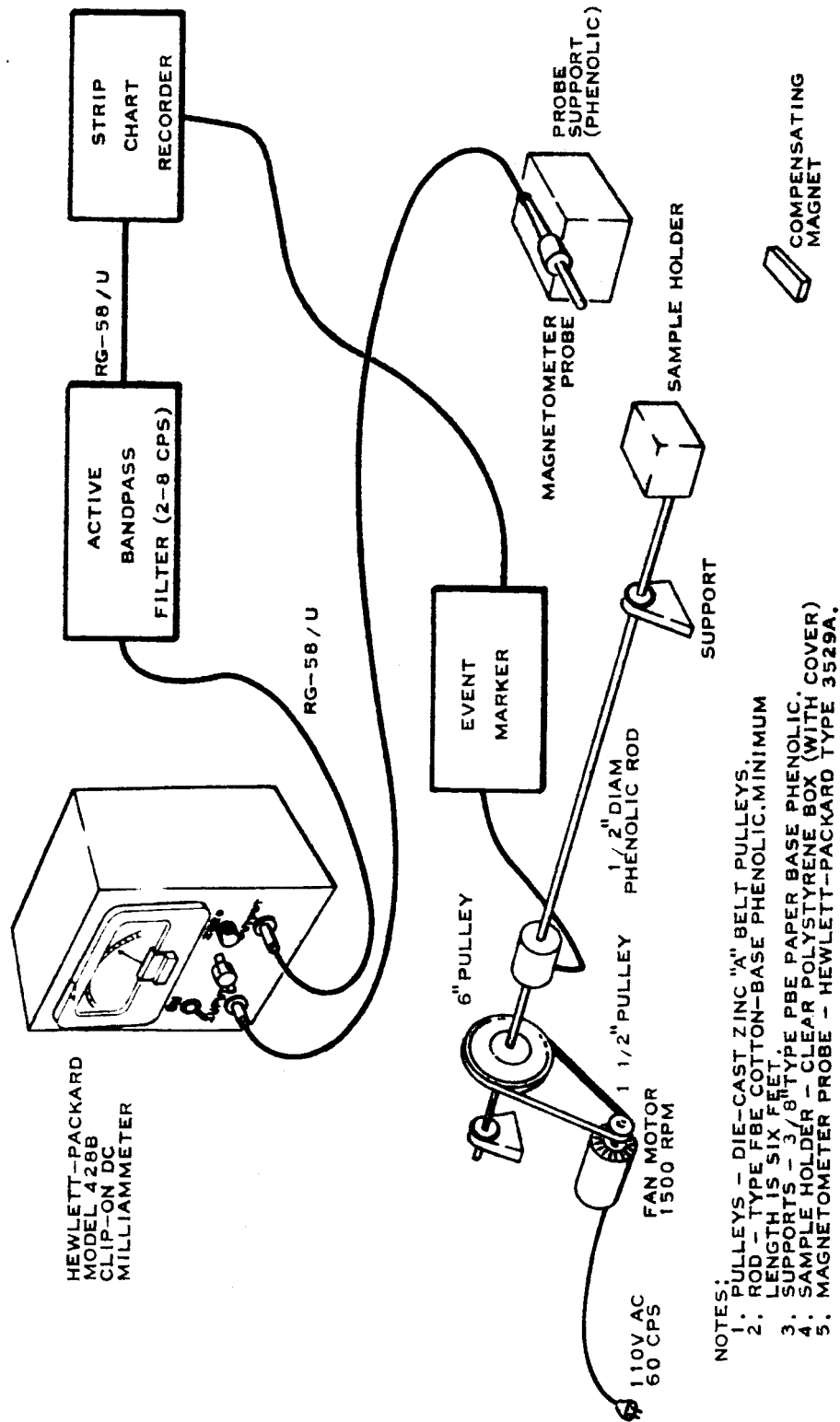


Figure 3. Component Measurement Test Setup

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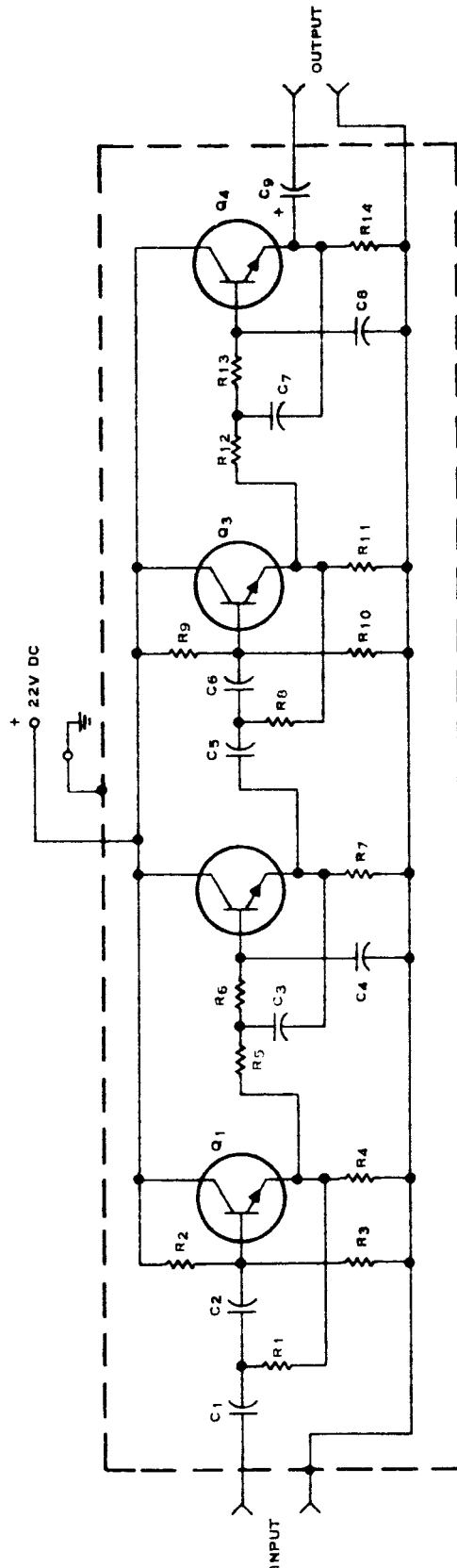
The conversion factor of the 3529A probe is 1:1, producing a reading from the 428B in milliamperes directly equal to the measured field strength in milligausses. Full-scale deflection on the 1-ma scale therefore corresponds to 100 gammas. Three spin axes are chosen for the component sample; these will be standardized for various component types. The sample is then rotated about each spin axis in turn, at approximately 5 cps. The magnetometer measures a sinusoidal change in field whose peak-to-peak value represents twice the magnitude of the field due to an equivalent fixed dipole aligned perpendicular to the spin axis and parallel to the magnetometer probe when the peak reading occurs.

The sensitivity of the flux-gate is limited to 1 gamma by the meter scale of the 428B. This may be appreciably improved by recording the magnetometer output on a high-sensitivity recorder. A Sanborn Model 150 with Stabilized DC Preamplifier Model 150-1800 was used in our investigation. Sensitivity in excess of 0.1 gamma was achieved with this apparatus. A variety of high impedance (100 kilohms or greater) and high sensitivity (1 millivolt or less per minor division of 50-division chart paper) recorders are also usable. A single-channel recorder may be used if it has provisions for remote marker application (as does the Sanborn Model 150); otherwise, a two-channel recorder should be used. The recorder should have adjustable sensitivity and pen centering and should have flat amplifier and pen response beyond 5 cps. Acceptable recorders include the Texas Instruments oscillo/riter,* Sanborn Model 320, Brush Mark 280, and Honeywell Model 153X16 instruments.

The bandpass filter inserted between the magnetometer output and the recorder input is shown schematically in Figure 4. The filter has a bandpass of 2 to 3 cps. The response at each end of the bandpass falls off at approximately 24 decibels per octave and is down 47 decibels at 60 cps. This bandpass was chosen because it represents the region of minimum background noise centered about the 5-cps rotation frequency. If miniature nonpolar tantalum capacitors are used (such as made by Components, Inc.), the filter can easily be built in a small minibox. Since the filter has a midband insertion loss of approximately 2 decibels, the recorder sensitivity must be increased somewhat to compensate for the signal loss. This is covered in the calibration steps contained in the next section.

Since it is nearly impossible to perfectly orient the magnetometer probe perpendicular to the ambient field, a compensating magnet is used to produce a static magnetic field sufficient to allow the 428B to operate on its most sensitive scale. Ideally, the meter should be adjusted by means of the compensating magnet and its own zero control to read approximately midscale. This permits measurement of a ± 50 -gamma change in field intensity. A small meter magnet works quite well as a compensator.

*Trademark of Texas Instruments Incorporated.



Q1 - Q4	- 2N830	C1, C2, C5, C6	- 1μF
R1	- 110K	C3, C7	- 0.22μF
R2, R3, R13	- 240K	C4, C8	- 0.047μF
R4, R7, R11, R14	- 470K	C9	- 1μF, 35V DC
R5	- 36K		
R6	- 510K		
R8	- 82K		
R9, R10	- 430K		
R12	- 56K		

NOTES:

1. ALL RESISTORS 1/4 WATT.
2. CONNECTORS ARE INSULATED BANANA JACKS.
3. C1-C8 ARE ±5% NONPOLAR TANTALUM CAPACITORS.
30V DC MINIMUM.
4. ENCLOSURE: MINIBOX OR OTHER SUITABLE ENCLOSURE.

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Figure 4. Active Bandpass Filter, 2-8 CPS

An event marker switch is mounted on the spin fixture so recording of a point on the sinusoidal chart trace can be correlated with a mechanical position of the fixture. The event marker may be a microswitch that is tripped twice per revolution of the rod. The switch controls a battery that applies a marker to the chart recorder each time the switch is tripped.

One advantage of this test setup is that it is self-calibrating. Calibration is performed by rotating a small magnet or a magnetic component, noting the meter deflection on the 428B, and adjusting the recorder sensitivity for the equivalent deflection. Since the recorders mentioned previously have precision attenuators, scale changes can be made by changing the recorder preamplifier attenuator setting.

The major precaution to be taken with this method is to avoid local disturbances (such as moving meters or large pieces of predominantly steel equipment) within approximately 15 feet of the magnetometer probe. These disturbances will cause the 428B meter to deflect off scale. This will not harm the instrument, since its circuitry automatically limits on overloads. Such a disturbance will, however, temporarily disrupt the output signal of the 428B to the recorder.

b. Test Procedure

- (1) Set up and connect equipment as in Figure 4.
- (2) Turn on the Hewlett-Packard 428B, the recorder, and the filter power. Set the 428B RANGE switch to the 300-ma position. Allow a 5-minute warmup period.
- (3) Orient the Hewlett-Packard 3529A magnetometer probe as nearly perpendicular to the ambient magnetic field as possible by adjusting the probe position for the lowest on-scale reading possible on the 428B. The RANGE switch is changed to progressively lower current scales as this is done.
- (4) Adjust the compensating magnet and the 428B ZERO control for a reading of approximately 0.5 milliamperes with the RANGE switch in the 1-ma position. Full scale on the meter equals 100 gammas.

CAUTION

The operator should remove his wristwatch and any magnetic rings, etc., while performing steps (3) and (4).

DATE <u>6/2/64</u>		OPERATOR <u>G.B. Smith</u>		DATE EQUIP. LAST CALIBRATED <u>6/1/64</u>		REMARKS																								
<p>COMPONENT SKETCH SHOWING AXES OF ROTATION</p> <p>LEAD LENGTH <u>.25 inch</u></p>						<p>TEST DISTANCE (INCHES)</p> <p>12</p>		<p>GAMMAS (P-P)</p> <table border="1"> <thead> <tr> <th>X AXIS</th> <th>Y AXIS</th> <th>Z AXIS</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>8</td></tr> <tr><td>0</td><td>0</td><td>7.4</td></tr> <tr><td>2</td><td>2.4</td><td>1.5</td></tr> <tr><td>0</td><td>0</td><td>8.2</td></tr> <tr><td>0</td><td>0.1</td><td>9.4</td></tr> <tr><td>0</td><td>0</td><td>6.3</td></tr> </tbody> </table>		X AXIS	Y AXIS	Z AXIS	0	0	8	0	0	7.4	2	2.4	1.5	0	0	8.2	0	0.1	9.4	0	0	6.3
X AXIS	Y AXIS	Z AXIS																												
0	0	8																												
0	0	7.4																												
2	2.4	1.5																												
0	0	8.2																												
0	0.1	9.4																												
0	0	6.3																												
COMPONENT IDENT		NR		RUN		<p>1 1 1 2 2 2 3 3 3 4 4 4</p>																								
CAPACITOR		Tantalum, Solid		SCM105FP006A2		<p>Mfg. date code 1971</p> <p>Lead color different from rest of sample.</p>																								
COMPONENT CLASS		TYPE		PART NUMBER		MANUFACTURER																								
16635		Tantalum, Solid		SCM105FP006A2		Texas Instruments Inc. P.O. Box 5012, Dallas, Texas 75222																								

IF THE COMPONENT IS MAGNETIC:

(1) WHAT PARTS ARE MAGNETIC? Leads

(2) WHAT IS MATERIAL USED? Copperweld, except #3 apparently nickel

(3) WHAT MODIFICATIONS ARE REQUIRED FOR ACCEPTABILITY? Change lead material to alloy 180

Figure 5. Typical Data Sheet

- (5) Calibrate the recorder to 100 gammas full scale by manually rotating a small magnet or magnetic component in the sample holder at a sufficient distance from the probe to give a 50-gamma (0.5 ma) meter deflection. Then rotate the magnet at the same distance at 5 cps. Adjust the recorder sensitivity and attenuator settings for a half-scale deflection.
- (6) Move the sample holder so that the geometric center of the sample holder is a standardized test distance from the probe. Increase the recorder sensitivity to 1 gamma full scale. Operate the spinner mechanism with no component sample in the sample holder. There should be no detectable magnetic signal due to the spinner.
- (7) Place a component in the sample holder with one of its test axes aligned with one event marker trip lever. Secure the component in this position with nonmagnetic foam rubber or plastic.
- (8) Rotate the component at 5 cps.
- (9) Record on the data sheet the peak-to-peak magnetic field intensity between two adjacent event marks.

NOTE

The event markers are 180 degrees apart.

- (10) Repeat steps (7), (8), and (9) with the component position changed within the sample holder so that the component is rotated about each of the specified spin axes.

c. Cataloging the Data

Measurement data will be tabulated on standardized test data sheets such as that shown in Figure 5. These sheets may then be cataloged for ease of reference. The data sheet may be modified as a result of the magnetization stability study at the beginning of the program.

C. Analyzing and Using the Data

The most immediate results of the measurement program will be lists of magnetically clean components, i.e., components that exhibit no equivalent dipole moment even after magnetization. There will also be components which are magnetic, for which manufacturer data indicates one small part to be the offender. If there are no other substitute components made by a different manufacturer which are nonmagnetic, the manufacturer may be

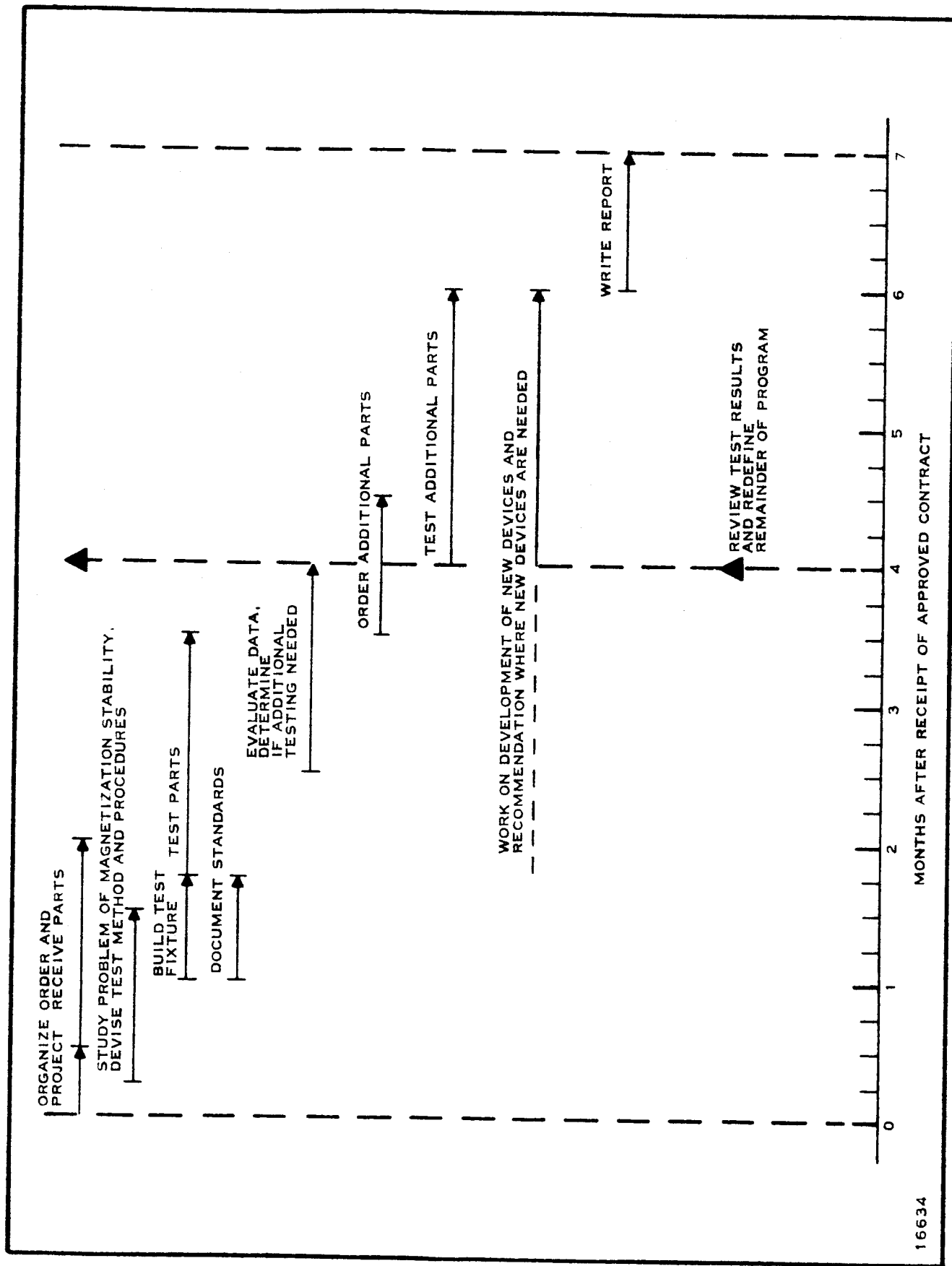


Figure 6. Suggested Schedule for Measurement Program

contacted to determine if the magnetic part or material can be replaced. A decision to develop the modified component can then be made based on the difficulty of the modification and the magnitude of its equivalent dipole moment.

When a number of identical components which exhibit equivalent dipole moments are tested, a statistical distribution will be observed. This distribution will allow a specification for the component to be written with a degree of certainty about the yields expected. The distribution may also be used to give some practical limits and values to the statistical methods of defining spacecraft magnetic environment described in the next section. Initial testing may indicate a distribution with a large variance which requires more units to be tested in order to completely define the distribution. As distributions for various components are defined, it may be possible to set up classifications or categories which define a large group of components. These classifications could be based on either magnetic field intensity magnitude or variance characteristics. A spacecraft might then be divided into zones in which only certain component classes are allowed within a specified distance from the magnetometer. This possibility, of course, cannot be explored until some data is accumulated.

Another important application of this data will be its use with a sampling procedure such as that defined in MIL-STD-105D to form a guide for manufacturers to follow in the future in terms of setting specifications, testing and quality control. The data will aid in setting industry-wide standards with a degree of certainty of the practicality and usefulness of these standards.

D. Test Program Schedule

A suggested schedule for the measurement program is diagrammed in Figure 6. Two months are allotted for organizing the program, ordering and receiving the parts listed in Appendix A. During this time, the project will be staffed, manufacturer questionnaire and cover letter finalized, and purchase orders written.

Five weeks are allotted to study the problem of magnetization stability. This will include studying the basic phenomena and environments that affect them. Practical values for the limits of the environments (magnetizing force, temperature, vibration level, etc.) must be established. Considering all these factors, test methods and procedures must then be formulated. In order to accomplish this in the specified time, personnel already familiar with the problem will be required.

By the end of the seventh week, test fixtures must be fabricated and test procedures and data sheets finalized. The remainder of the period will be spent testing, analyzing test data, determining where additional tests are required, and working with manufacturers to modify standard devices by removing magnetic parts and materials.

It is estimated that a significant program can be accomplished in 7 months. However, it should be realized that testing and developing nonmagnetic parts could continue indefinitely depending upon how complete a preferred parts list of nonmagnetic components is desired and how complete a list is readily available from standard components. Therefore, it is recommended that, at the end of the fourth month, the results to that date be reviewed and the remainder of the program be redefined.

The measurement program results should include:

1. A recommended preferred parts list indicating components which are nonmagnetic and those which are not. This will include necessary manufacturing control data.
2. Test data cataloged and indexed for easy reference.
3. A list of parts which could be made nonmagnetic with some modification and details of the modification.

III. A STOCHASTIC APPROACH TO THE PROBLEM OF ALLOWABLE MAGNETIC MOMENTS IN A SPACE VEHICLE

A. Scope of the Problem and an Approach to Its Solution

In space vehicles containing magnetometer experiments, care must be taken that the magnetic sensor is not disturbed by magnetic fields due to other electronic or mechanical components within the spacecraft.

The problem of placement of a few simple components of known magnetic moments is fairly straightforward. The problem of designing a complex electronic assembly that must be placed within a certain distance of the sensor and yet be magnetically acceptable is more difficult. The designer must assemble hundreds of items, which are probably magnetic, and guarantee that their net effect at the sensor will not exceed some prescribed limit.

It is to the problem of establishing initial component selection and configuration guidelines and checks that the stochastic or probabilistic approach is directed. This approach will permit the designer to ascertain with some confidence the probable magnetic effect at the sensor of N magnetic moments whose amplitudes, locations, and directions are described by probability density functions. Conversely, given a set of probabilistic limits on the allowable magnetic field at some distance from the unit to be designed, he may ascertain the permissible distribution function of the magnetic moments.

The statistical approach frees the designer from making a precise field calculation for each component and each possible configuration. In return for this unburdening, he must accept more general answers—answers which are statistical rather than precise.

This preliminary study describes the probability density functions for three magnetic field components at a sensor in terms of random variables describing the position, orientation, and magnetic moment of a statistical dipole in a spherical coordinate system. The resultant probability density functions are expressed as multiple integrals of functions of these random variables and the Jacobian of the transformation.

Determination of these probability density functions describing the field components for a particular configuration of the sensor and electronics package requires that the probability density function describing position, orientation, and magnetic moment of the n^{th} dipole be known. Since such factors as preferential alignment of components or moments and regions of component clustering will significantly affect the results, it is important that these input probability density functions be accurately determined.

The study indicates further how the result for the n^{th} dipole may be extended to a statistical description for N such dipoles.

It is further indicated that, given a probabilistic description of the magnitude of the allowed field components at the sensor and probabilistic descriptions of the other variables, the problem may be inverted. This inversion might obtain, for example, a probabilistic description or a histogrammic description of the allowed range and distribution of the N individual dipole moments.

In order to illustrate the technique, the probability density function for one of the field components is determined for the special case of a spherical electronics package located at a distance, r_0 , from the sensor and containing N dipoles, whose magnetic moments are uniformly distributed but preferentially oriented in a 60° cone about the vertical axis. The problem is then inverted to show how the allowed range of the magnetic moments is determined by the limits set on the magnitude of one of the field components.

As a further illustration, the case of a truncated spherical package (approximating a cylindrical electronics package) is considered and the method of solution and inversion indicated.

In order to show the broad application of the stochastic, or probabilistic, approach, the method is also applied to a magnetic stability problem. Thus, instead of considering probabilistic descriptions of the spatial configuration of a set of dipoles, the case of dipoles whose magnetic characteristics change in a probabilistic way with time is treated.

B. Introduction

In space vehicles containing magnetometer experiments care must be taken that the magnetic sensor is not disturbed by magnetic fields due to other electric, electronic, or mechanical components within the spacecraft.

For instance, a single iron core transformer placed 3 feet from the magnetometer might produce a larger effect at the sensor than the Martian magnetic field at a distance of 3 Martian radii. These unwanted fields may be reduced to acceptable levels at the sensor by cancelling them out through compensation techniques or by the obvious stratagem of limiting the size of magnetic moments permitted within certain distances from the sensor.

For the case of a few simple components whose magnetic moments are well known, the problem of how close to the sensor they may be placed is fairly straightforward. However, the problem of designing a complex electronic assembly that must be placed within a certain distance of the sensor and yet must not produce a magnetic field above some small value is considerably more difficult. In this instance the designer must assemble together hundreds of items such as transistors, resistors, transformers, chokes, lugs, shafts, etc., all of which are probably magnetic, and guarantee a priori that their net effect at the sensor will not exceed some prescribed limit. The problem is further complicated by the fact that some of the components and their magnetic moments will be more or less randomly oriented, while others will be aligned in preferred directions (for example, resistors on circuit boards).

Obviously, the designer could build the unit, measure the magnetic field produced, and through a "cut and try" sequence, eventually converge on a magnetically acceptable design.

An alternate approach is to combine empirical measurements and knowledge of the physical problem into a stochastic or statistical model from which guidelines may be established for making initial component selection. This approach will permit the designer of a very complex unit to ascertain with some confidence the probable magnetic effect at the sensor of N magnetic moments whose amplitudes, locations, and directions are described by probability density functions.* Conversely, given a set of probabilistic limits on the magnetic field at some distance from the unit to be designed, he may ascertain the permissible parameters of position, orientation, and magnetic moments of individual components or assemblies.

The purpose of this study was to investigate the feasibility of deriving such a statistical model incorporating the maximum information available. This would allow decision makers, such as the advanced system designers and the component engineers, to make those decisions, which are optimum in a statistical sense, and to provide a measure of confidence in the decisions.

*See "Explanation of Statistical Terms," Appendix B at the end of this report.

C. Objectives

1. Given the coordinate location of an electronics package which contains N dipoles whose position, orientation, and magnitude are given by probability density functions, derive the probability density function of a component of the magnetic field intensity at a sensor outside the package.
2. Given allowed probability density functions of the component of the field intensity, it is desired to be assured within some probability that the absolute value of the field intensity will not exceed some prescribed value. From the model one should be able to determine in what manner the original parameters should be adjusted.

As an example, one could specify for arbitrary distributions of dipole directions and positions, the allowed distributions of the sizes of the magnetic moments. This distribution function of sizes of allowed moments could be quantitized into a histogram for use by the package designer.

D. Introduction to the Derivation

The effect of each dipole upon the magnetic sensor will be a function of its position within the three dimensional electronics package, the position of the sensor relative to the electronics package, the orientation of the dipole, and the magnitude of the magnetic moment. Of these parameters, only the position of the sensor with respect to the electronics box will be exactly determined; the other parameters can be expressed as random variables and can be described by probability density functions. The components of the magnetic field intensity as seen by the sensor can be considered as functions of several random variables and can also be described by probability density functions.

The problem is then to transform the probability density functions of the position, orientation, and magnitude of each dipole into the probability density function of the magnetic field intensity. This transformation will be performed for the effect of one dipole with random position, orientation, and magnitude, and then extended to include the effect of N dipoles, where N is a very large number.

E. Derivations of Probability Density Functions

The magnetic field intensity of a dipole may be expressed in any of the three conventional coordinate systems: rectangular, cylindrical, or spherical. The field equations were examined in each of the coordinate systems. It was found that the problem of transforming multiple probability density functions into one could be simplified by operating in the spherical coordinate system. Equations (1) and (2) express the radial and tangential components of the far field intensity in spherical coordinates for a centered dipole aligned along the Z axis (see Figure 7).

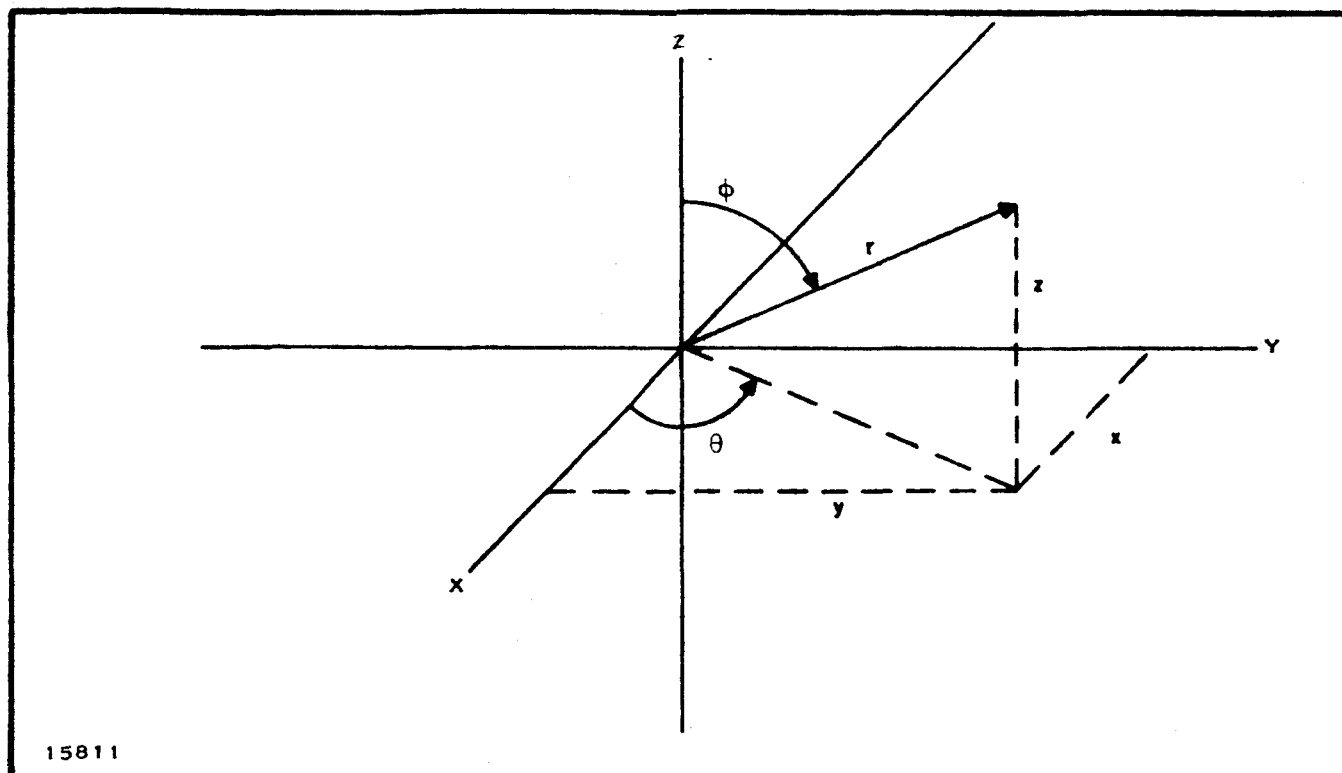


Figure 7. The Spherical Coordinate System

$$H_r = \frac{2m \cos \phi}{r^3} \quad (1)$$

$$H_\phi = \frac{m \sin \phi}{r^3} \quad (2)$$

In the above equations m expresses the magnitude of the magnetic moment. Coordinates r , ϕ , and θ are defined in Figure 1. The components of field intensity may be resolved into x , y , and z directions of a rectangular coordinate system by the following relations:

$$H_x = H_r \sin \phi \cos \theta + H_\phi \cos \phi \cos \theta \quad (3)$$

$$H_y = H_r \sin \phi \sin \theta + H_\phi \cos \phi \sin \theta \quad (4)$$

$$H_z = H_r \cos \phi - H_\phi \sin \phi \quad (5)$$

Substituting H_r and H_ϕ from Equations (1) and (2), H_x , H_y , and H_z become

$$H_x = \frac{3m}{2r^3} \sin 2\phi \cos \theta \quad (6)$$

$$H_y = \frac{3m}{2r^3} \sin 2\phi \sin \theta \quad (7)$$

$$H_z = \frac{m}{2r^3} (3 \cos 2\phi + 1) \quad (8)$$

The components of the magnetic field intensity have been resolved into rectangular coordinates for compatibility with the coordinate system of the sensor; the variables of the equations have been retained in spherical coordinates to simplify the mathematics.

Let ϕ_0 , θ_0 , and r_0 be the coordinants which relate the center of the electronics box to the sensor; ϕ_1 , θ_1 , and r_1 be the coordinants which translate the dipole with respect to the center of the electronics box; and ϕ_2 and θ_2 be the coordinants which rotate the axis of the dipole with respect to the coordinant system. In Equations (6), (7), and (8):

$$r = r_1 - r_0 \quad (9)$$

$$\phi = \phi_1 + \phi_2 - \phi_0 \quad (10)$$

$$\theta = \theta_1 + \theta_2 - \theta_0 \quad (11)$$

The parameters r_0 , ϕ_0 , and θ_0 are constants. Since each dipole may be randomly positioned and randomly oriented within the electronics package, the parameters ϕ_1 , θ_1 , r_1 , ϕ_2 , and θ_2 may be considered as random variables which are described by probability density functions.

Since the random variables ϕ_1 and ϕ_2 occur in a sum, they may be combined as a new random variable Φ ; likewise, θ_1 and θ_2 may be combined as Θ . The probability density function of Φ is then the convolution of the probability density functions of ϕ_1 and ϕ_2 (Appendix B Statistical Terms), and the probability density function of Θ is the convolution of the probability density functions of θ_1 and θ_2 . The magnitude, m , of the magnetic moment may also be considered as a random variable.

A component of the magnetic field intensity, such as H_x , may now be expressed as a function of four random variables m , r_1 , Φ , and Θ ; therefore, H_x is also characterized by a probability density function. Let the probability density functions of H_x , m , r_1 , Φ , and Θ be designated as $p(H_x)$, $p(m)$, $p(r_1)$, $p(\Phi)$, and $p(\Theta)$, respectively. Let the joint probability density function of m , r_1 , Φ , and Θ be designated as $p(m, r_1, \Phi, \Theta)$. THE PROBLEM THEN BECOMES TO TRANSFORM THE JOINT PROBABILITY DENSITY FUNCTION $p(m, r_1, \Phi, \Theta)$ INTO $p(H_x)$. For each value of m , r_1 , Φ , and Θ , there is one and only one corresponding value of H_x .

There are two standard approaches¹ available for obtaining $p(H_X)$ from $p(m, r_1, \Phi, \theta)$. The first approach is indicated by Equation (12).

$$p(H_X) = \frac{d}{dH_X} \int \int \int \int_S p(m, r_1, \Phi, \theta) dS \quad (12)$$

where the integration is performed over the sample space S , which is defined by the orthogonal basis (m, r_1, Φ, θ) , producing the probability distribution function of H_X . The differentiation with respect to H_X results in the probability density function $p(H_X)$. In this problem, this approach leads to difficult elliptic integrals.

The second approach involves a direct transformation of the joint probability density function of m, r_1, Φ , and θ to a new joint probability density function of four new variables, one of which is H_X . The function $p(H_X)$ is then obtained by integrating the new joint density function over the limits of the three extraneous variables. This was the method used to obtain $p(H_X)$.

When N random variables are related to N new random variables by a one-to-one mapping, their joint probability density functions may sometimes be equated by means of a Jacobian (see Appendix B). For example, if y_1, y_2, y_3 , and y_4 represent the new variables which are defined as single-valued, continuous functions of the old variables, x_1, x_2, x_3 , and x_4 , then the joint probability density functions may be related as

$$p(y_1, y_2, y_3, y_4) = p(x_1, x_2, x_3, x_4) |J| \quad (13)$$

where J is the Jacobian of the transformation and is defined by the expression¹

$$J = \frac{\partial(x_1, x_2, x_3, x_4)}{\partial(y_1, y_2, y_3, y_4)} \quad (14)$$

In the above problem, the four variables m, r_1, Φ , and θ are to be transformed into one component such as the variable H_X ; therefore, in order to determine a Jacobian, three additional variables must be introduced. The selection of these three variables is somewhat arbitrary under the constraints of being single-valued, continuous functions of m, r_1, Φ , and θ . A particular choice of these additional variables and their relation to the old variables are given below.

$$u = r_1 \quad (15)$$

$$v = \theta \quad (16)$$

$$w = \Phi \quad (17)$$

¹ See page 35, Reference 1.

The joint probability density function of the new random variables H_x , u , v , and w may be written as

$$p(H_x, u, v, w) = p(m, r_1, \Phi, \theta) \left| \frac{\partial(m, r_1, \Phi, \theta)}{\partial(H_x, u, v, w)} \right|. \quad (18)$$

The old variables may also be expressed as functions of the new.

$$m = f_1(H_x, u, v, w) = \frac{2H_x(u - r_0)^3}{3 \sin 2(w - \phi_0) \cos(v - \theta_0)} \quad (19)$$

$$r_1 = f_2(H_x, u, v, w) = u \quad (20)$$

$$\theta = f_3(H_x, u, v, w) = v \quad (21)$$

$$\Phi = f_4(H_x, u, v, w) = w. \quad (22)$$

The Jacobian may be written in determinant form

$$J_x = \begin{vmatrix} \frac{\partial f_1}{\partial H_x} & \frac{\partial f_2}{\partial H_x} & \frac{\partial f_3}{\partial H_x} & \frac{\partial f_4}{\partial H_x} \\ \frac{\partial f_1}{\partial u} & \frac{\partial f_2}{\partial u} & \frac{\partial f_3}{\partial u} & \frac{\partial f_4}{\partial u} \\ \frac{\partial f_1}{\partial v} & \frac{\partial f_2}{\partial v} & \frac{\partial f_3}{\partial v} & \frac{\partial f_4}{\partial v} \\ \frac{\partial f_1}{\partial w} & \frac{\partial f_2}{\partial w} & \frac{\partial f_3}{\partial w} & \frac{\partial f_4}{\partial w} \end{vmatrix}. \quad (23)$$

By evaluating the partial derivatives of the last three columns, the following form is derived:

$$J_x = \begin{vmatrix} \frac{\partial f_1}{\partial H_x} & 0 & 0 & 0 \\ \frac{\partial f_1}{\partial u} & 1 & 0 & 0 \\ \frac{\partial f_1}{\partial v} & 0 & 1 & 0 \\ \frac{\partial f_1}{\partial w} & 0 & 0 & 1 \end{vmatrix}. \quad (24)$$

Expanding by minors yields

$$J_x = \frac{\partial f_1}{\partial H_x} = \frac{2(u - r_0)^3}{3 \sin 2(w - \phi_0) \cos (v - \theta_0)} \quad (25)$$

Therefore,

$$p(H_x, u, v, w) = (m = f_1, r_1 = f_2, \Phi = f_3, \phi = f_4) \left| \frac{2(u - r_0)^3}{3 \sin 2(w - \phi_0) \cos (v - \theta_0)} \right| \quad (26)$$

From this expression, the unconditional probability density function of H_x may be obtained by the integration of the joint probability density function, $p(H_x, u, v, w)$, with respect to the random variables u, v , and w over their entire range of possible values. Thus,

$$p(H_x) = \int \int \int p(f_1, u, v, w) \left| \frac{2(u - r_0)^3}{3 \sin 2(w - \phi_0) \cos (v - \theta_0)} \right| dw dv du \quad (27)$$

Expressions for the other components of the magnetic field intensity may be obtained in a like manner. The Jacobians for transformations to $p(H_y)$ and $p(H_z)$ are given by Equations (28) and (29), respectively.

$$J_y = \frac{2(u - r_0)^3}{3 \sin 2(w - \phi_0) \sin (v - \theta_0)} \quad (28)$$

$$J_z = \frac{2(u - r_0)^3}{3 \cos 2(w - \phi_0) + 1} \quad (29)$$

F. The Case of Assumed Probability Density Functions of the Random Variables

The probability density functions of H_x, H_y , and H_z as defined above cannot be further evaluated unless assumptions are made concerning the joint probability density functions of the random variables m, r_1, Φ , and ϕ . Preferably, such assumptions would only be made after considering histograms of empirical data obtained from an extensive measurements program. Since such data is not available at this time, simple assumptions will be made concerning the probability density functions in order to demonstrate computational procedures and to investigate possible difficulties which might arise in integration.

G. Example for a Spherical Package

Assume that the magnitude of the magnetic moment is uniformly distributed from 0 to M . Since the integral of the probability density function of $m, p(m)$, must be equal to unity, we may solve for the amplitude, A , of $p(m)$ from the following relation.

$$\int_0^M A \, dm = A(M - 0) = 1; \quad (30)$$

therefore,

$$A = \frac{1}{M} \quad (31)$$

$$p(m) = \begin{cases} \frac{1}{M} & \text{for } 0 \leq m \leq M \\ 0 & \text{elsewhere} \end{cases} \quad (32)$$

Assume that all dipoles are uniformly distributed within a spherical package of radius, R.

$$p(r_1) = \begin{cases} \frac{1}{R} & \text{for } 0 \leq r_1 \leq R \\ 0 & \text{elsewhere} \end{cases} \quad (33)$$

$$p(\phi_1) = \begin{cases} \frac{1}{\pi} & \text{for } 0 \leq \phi_1 \leq \pi \\ 0 & \text{elsewhere} \end{cases} \quad (34)$$

$$p(\theta_1) = \begin{cases} \frac{1}{2\pi} & \text{for } 0 \leq \theta_1 \leq 2\pi \\ 0 & \text{elsewhere} \end{cases} \quad (35)$$

Assume that the effective alignment of all dipoles is uniformly distributed from 0 to 60 degrees with respect to the Z axis.

$$p(\phi_2) = \begin{cases} \frac{3}{2\pi} & \text{for } 0 \leq \phi_2 \leq \frac{\pi}{3} \\ \frac{3}{2\pi} & \text{for } \frac{2\pi}{3} \leq \phi_2 \leq \pi \\ 0 & \text{elsewhere} \end{cases} \quad (36)$$

$$p(\theta_2) = \begin{cases} \frac{1}{2\pi} & \text{for } 0 \leq \theta_2 \leq 2\pi \\ 0 & \text{elsewhere} \end{cases} \quad (37)$$

The probability density function of $p(\theta)$ is obtained by the convolution of $p(\theta_1)$ and $p(\theta_2)$. The symbol * will henceforth denote convolution as in Equation (38).

$$p(\theta) = p(\theta_1) * p(\theta_2) \quad (38)$$

let

$$p(\theta_1) * p(\theta_2) = p_1(\theta) * p_2(\theta) ; \quad (39)$$

thus,

$$p(\theta) = \int_{-\infty}^{\infty} p_1(\theta) p_2(\theta - \theta) d\theta \quad (40)$$

by definition of the convolution integral. In the range $0 \leq \theta \leq 2\pi$,

$$p(\theta) = \int_{\theta-2\pi}^{\theta} \frac{1}{2\pi} \cdot \frac{1}{2\pi} d\theta = \frac{\theta}{4\pi^2} . \quad (41)$$

In the range $2\pi \leq \theta \leq 4\pi$,

$$p(\theta) = \int_{\theta-2\pi}^{2\pi} \frac{1}{2\pi} \cdot \frac{1}{2\pi} d\theta = \frac{1}{\pi} - \frac{\theta}{4\pi^2} . \quad (42)$$

Therefore,

$$p(\theta) = \begin{cases} \frac{\theta}{4\pi^2} & \text{for } 0 \leq \theta \leq 2\pi \\ \frac{1}{\pi} - \frac{\theta}{4\pi^2} & \text{for } 2\pi \leq \theta \leq 4\pi \\ 0 & \text{elsewhere} \end{cases} \quad (43)$$

In a similar manner $p(\Phi)$ may be obtained by convolving $p(\phi_1)$ and $p(\phi_2)$.

$$p(\Phi) = \begin{cases} \frac{3\Phi}{2\pi^2} & \text{for } 0 \leq \Phi \leq \frac{\pi}{3} \\ \frac{1}{2\pi} & \text{for } \frac{\pi}{3} \leq \Phi \leq \frac{2\pi}{3} \\ \frac{3\Phi}{2\pi^2} - \frac{1}{2\pi} & \text{for } \frac{2\pi}{3} \leq \Phi \leq \pi \\ \frac{5}{2\pi} - \frac{3\Phi}{2\pi^2} & \text{for } \pi \leq \Phi \leq \frac{4\pi}{3} \\ \frac{1}{2\pi} & \text{for } \frac{4\pi}{3} \leq \Phi \leq \frac{5\pi}{3} \\ \frac{3}{\pi} - \frac{3\Phi}{2\pi^2} & \text{for } \frac{5\pi}{3} \leq \Phi \leq 2\pi . \end{cases} \quad (44)$$

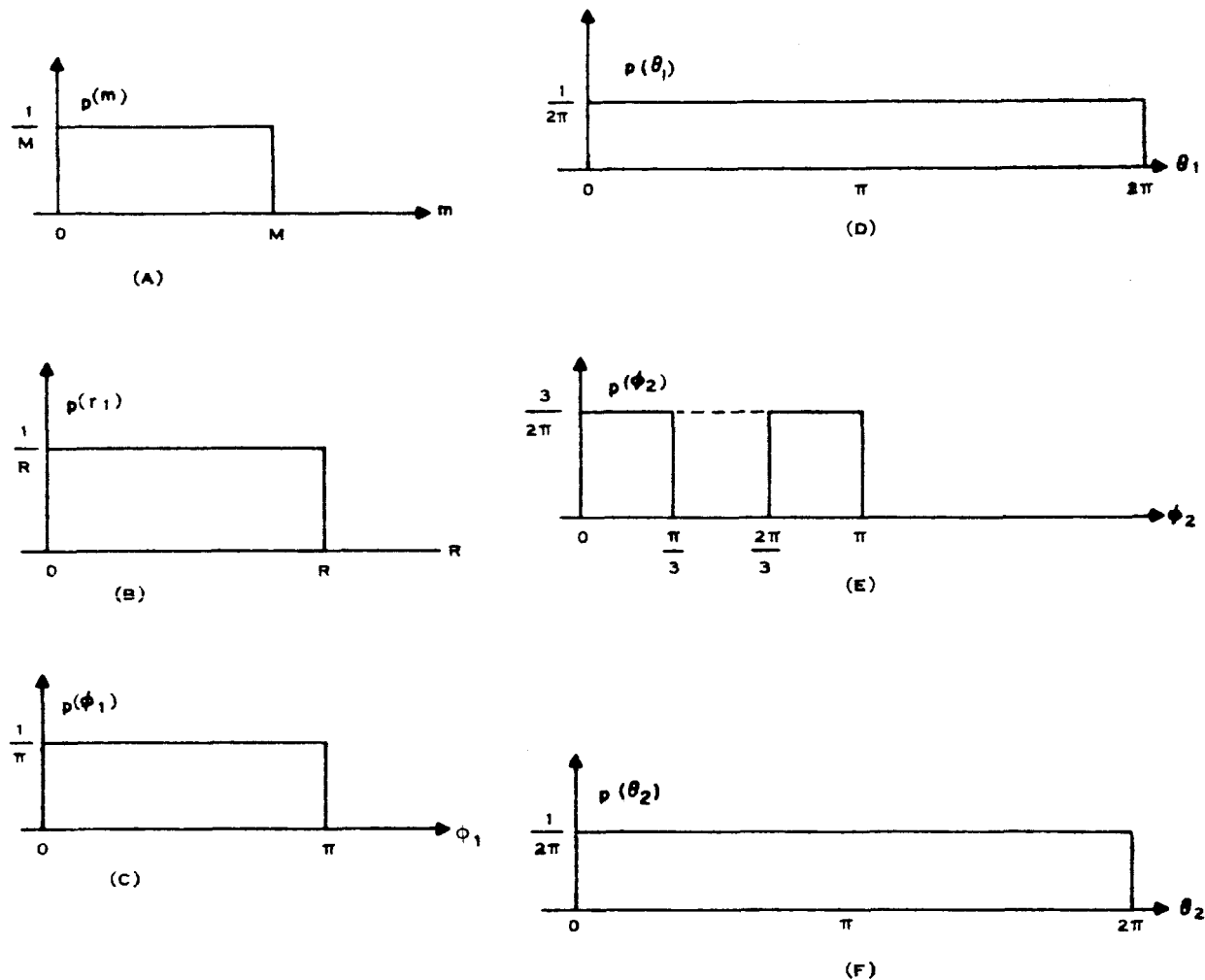


Figure 8. Probability Density Functions
 $p(m)$, $p(r_1)$, $p(\phi_1)$, $p(\phi_2)$, $p(\theta_1)$, $p(\theta_2)$

The probability density functions, $p(m)$, $p(r_1)$, $p(\phi_1)$, $p(\theta_1)$, $p(\phi_2)$, and $p(\theta_2)$ are shown in Figure 8. The probability density functions $p(\Phi)$ and $p(\Psi)$ are shown in Figure 9.

From the assumed conditions [Equations (32) through (37)], the random variables m , r_1 , ϕ_1 , θ_1 , ϕ_2 , and θ_2 are statistically independent. The random variables m , r_1 , Φ , and Ψ are also independent; therefore, their joint probability density function is the product of their individual probability density functions.²

$$p(m, r_1, \Phi, \Psi) = p(m) p(r_1) p(\Phi) p(\Psi). \quad (45)$$

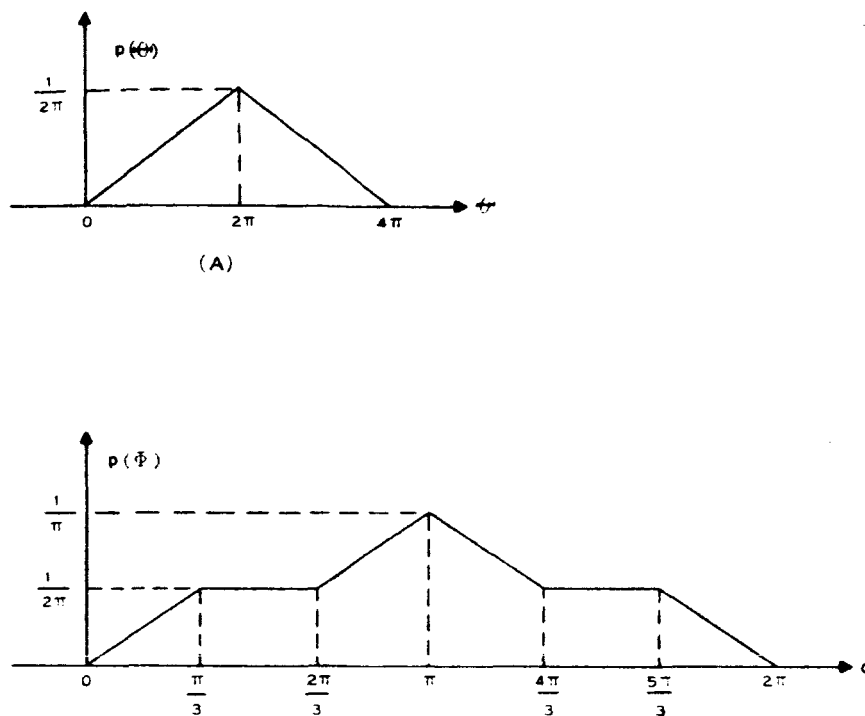


Figure 9. Probability Density Functions
 $p(\theta)$ and $p(\Phi)$

Equation (27) thus becomes

$$p(H_X) = \int \int \int p(m = f_1) p(r_1 = u) p(\theta = v) p(\Phi = w) |J_X| dw dv du. \quad (46)$$

In this case the order of integration is optional.

$$p(H_X) = \int_{w=0}^{2\pi} \int_{v=0}^{4\pi} \int_{u=0}^R \frac{p(v) p(w)}{MR} \left| \frac{2(u - r_0)^3}{3 \sin 2(w - \phi_0) \cos (v - \theta_0)} \right| du dv dw. \quad (47)$$

Integrating with respect to u ,

$$p(H_X) = \frac{|R^3 - 4R^2 r_0 + 6R r_0^2 - 4r_0^3|}{6M} \int_{w=0}^{2\pi} \int_{v=0}^{4\pi} \frac{p(v) p(w) dv dw}{|\sin 2(w - \phi_0) \cos (v - \theta_0)|}. \quad (48)$$

The last two integrals are improper integrals of the second kind and have not yet been evaluated in closed form. For particular ranges of values of v and w , the integrals may be expanded in an infinite series; this suggests the possibility of a numerical approximation on a digital computer.

It is not unreasonable to expect convergence since in this case, as will be shown, $p(H_x)$ must be constant and finite. Also, the integrals may possibly be solved analytically by techniques such as contour integration. The same type of integrals occur in the determination of the probability density functions of H_y and H_z . Since specific values of the integrals depend on particular constants ϕ_0 and θ_0 and assumed probability density functions of v and w , no general knowledge of the model could be obtained by a solution; therefore, laborious computation leading to results unique to the above assumptions were avoided. The solution to the remaining integrals will be represented symbolically by the constant K_x . Then

$$p(H_x) = \frac{K_x |R^3 - 4R^2r_0 + 6Rr_0^2 - 4r_0^3|}{6M} \quad (49)$$

Equation (49) describes the amplitude of the probability density function of H_x . In this case $p(H_x)$ is a constant. In general, the order of H_x in $p(H_x)$ will be equal to the order of m in $p(m)$ for all cases where m is statistically independent of the other random variables [see Equations (19) and (46)].

The assumed conditions provide a physical symmetry in terms of dipole orientation wherein positive and negative values of H_x are equally likely. The probability density function, $p(H_x)$, therefore has even symmetry about the origin and has a mean value of zero. By equating the area of $p(H_x)$ to unity, we may solve for the range of H_x . Where H_{x1} and H_{x2} are the respective minimum and maximum values of H_x ,

$$H_{x2} = -H_{x1} = \frac{3M}{K_x |R^3 - 4R^2r_0 + 6Rr_0^2 - 4r_0^3|} \quad (50)$$

NOTE

For a sensor located a great distance from the electronics package (that is, $r_0 \gg R$), the maximum magnitude of the magnetic field intensity is approximately proportional to the inverse cube of r_0 as one might expect.

Since r_0 is several times greater than R for most physical configurations, the effect of taking the absolute value in Equations (49) and (50) may be accomplished by changing the sign of all terms involving r_0 or R . Therefore,

$$p(H_x) = \begin{cases} \frac{K_x (4r_0^3 - 6Rr_0^2 + 4R^2r_0 - R^3)}{6M} & \text{for } H_{x1} \leq H_x \leq H_{x2} \\ 0 & \text{elsewhere} \end{cases} \quad (51)$$

The variance, V_x , of $p(H_x)$ may be expressed by

$$V_x = \int_{H_{x1}}^{H_{x2}} H_x^2 p(H_x) dH_x \quad (52)$$

$$V_x = \frac{3M^2}{K_x^2 (4r_0^3 - 6Rr_0^2 + 4R^2r_0 - R^3)^2} \quad (53)$$

H. Extension to N Dipoles

The above development was performed by considering one dipole randomly positioned and oriented with a random magnetic moment. When the number of dipoles is increased to N , where N is a very large number, the shape of the probability density function tends to become gaussian by the Central Limit Theorem.¹ For a large finite N , the curve will never become exactly gaussian; but such a shape will provide a good approximation. The tails of the theoretical gaussian curve extend to infinity; whereas the positive tails of the probability density function for N dipoles extend to N times the positive range of the density function for one dipole.

Assuming a gaussian curve for the probability density function of a component of the magnetic field intensity, such a curve is characterized by its mean and variance.

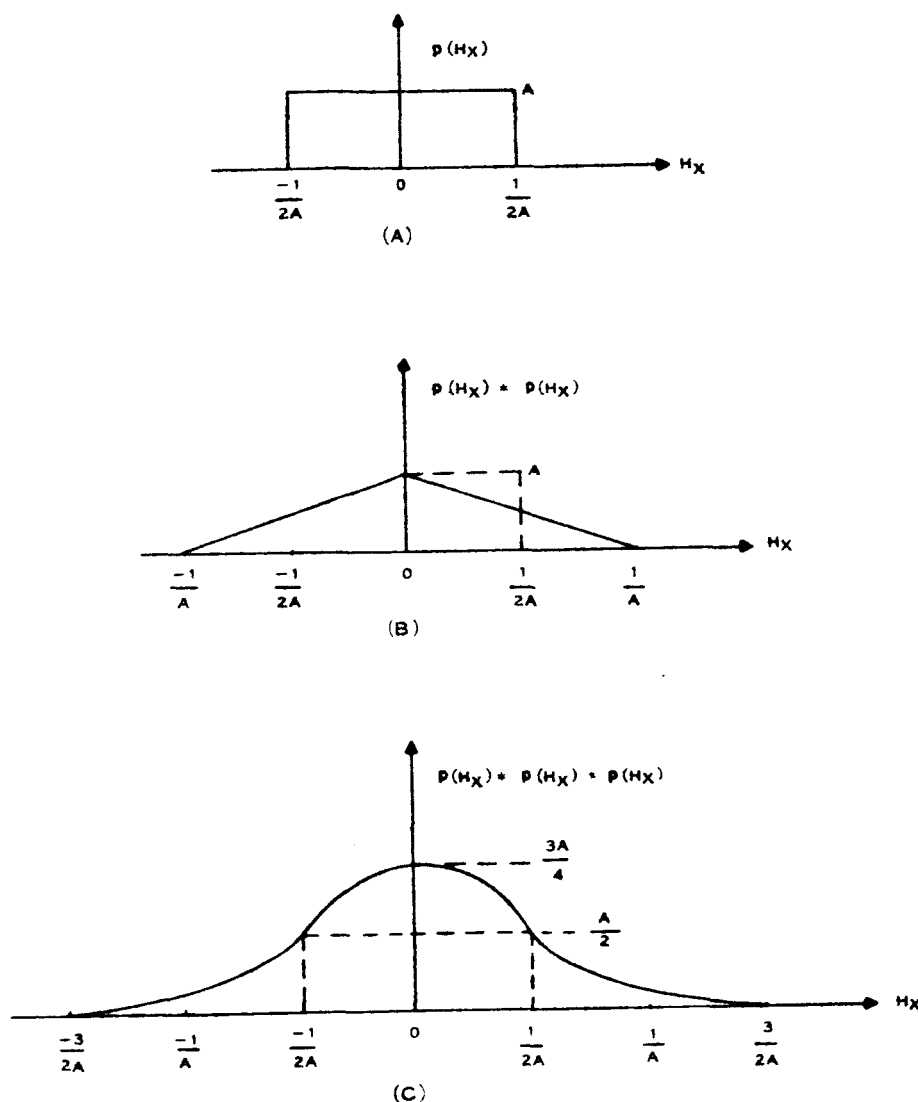
Components of the magnetic field intensity in the x direction from separate dipoles add algebraically. The probability density function for the sum of these components is the convolution of the probability density functions for the effect of the individual dipoles. Figure 10a shows a uniform probability density function for one dipole; Figure 10b shows the effect for two like dipoles; and Figure 10c shows the effect of three like dipoles. In Figure 10c the probability density function has already begun to assume the form of a gaussian type curve.

The gaussian curve for the effect of N dipoles will have N times the mean value of the probability density function for one dipole. The variance of this gaussian curve will be the variance of the density function for one dipole multiplied by N . Thus,

$$V_{x,N} = \frac{3M^2N}{K_x^2 (4r_0^3 - 6Rr_0^2 + 4R^2r_0 - R^3)^2} \quad (54)$$

where $V_{x,N}$ is the variance of the probability density function of H_x for the effect of N dipoles, each of which is described by several random variables having probability density functions of the form described above. This probability density function, $p(H_{x,N})$, where N is very large, will assume the form

$$p(H_{x,N}) = \frac{1}{\sqrt{2\pi V_{x,N}}} e^{-\left(\frac{H_x}{\sqrt{2 V_{x,N}}}\right)^2} \quad (55)$$



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Figure 10. Uniform Probability Density for One Dipole

I. Determination of Component Parameters from the Model

An alternate problem is given the above model of the gaussian probability density function, $p(H_X, N)$ having zero mean, to pick a range of permissible values of H_X and to choose a probability that a sample value of H_X chosen at random from the total population of H_X , will fall within the required limits.

As an example, suppose that the required probability that H_X lies between $-H$ and $+H$ is P . Recall that P is the area of $p(H_X)$ between $-H$ and $+H$. From a table of Areas of the Standard Normal Curve such as that of Pearson,³ a value Z_0 may be chosen corresponding to the value of P . Z_0 is a value of Z of the probability density function.

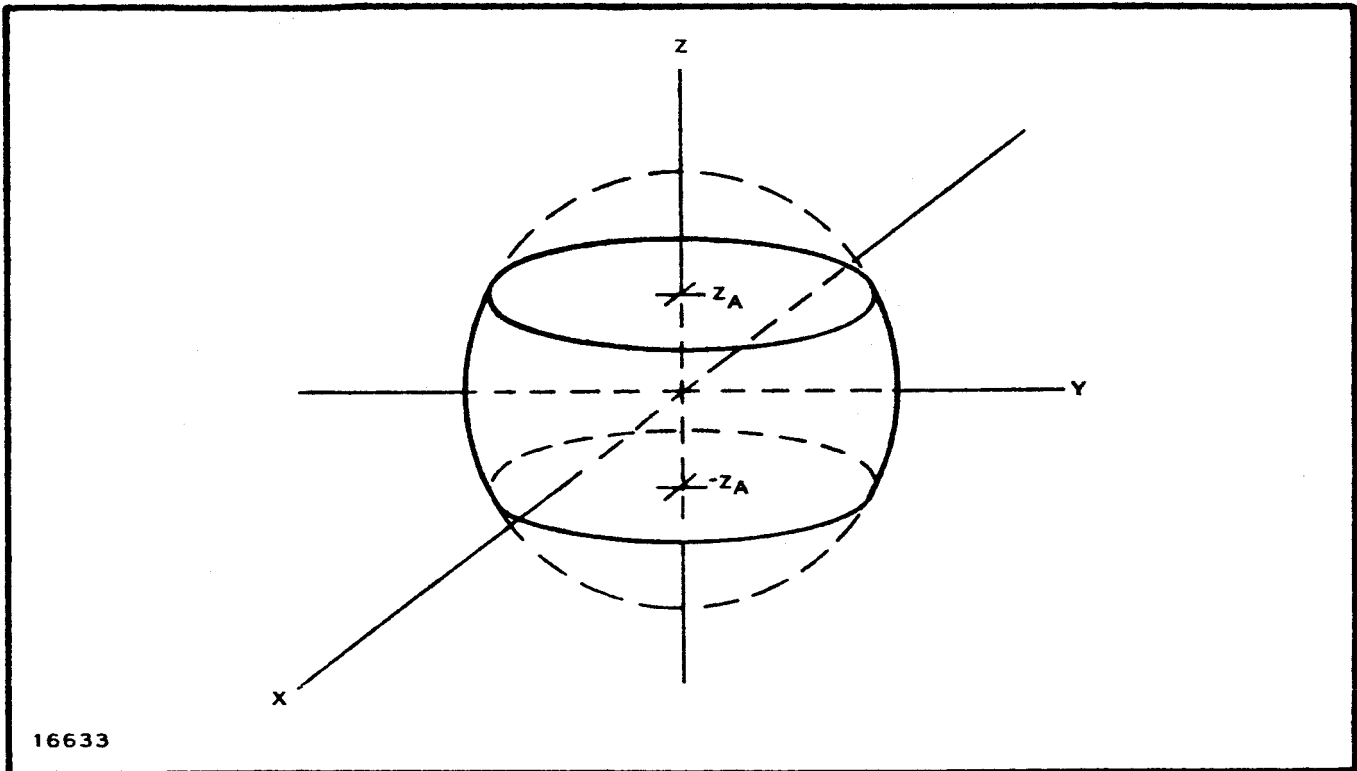


Figure 11. The Electronics Package as a Truncated Sphere

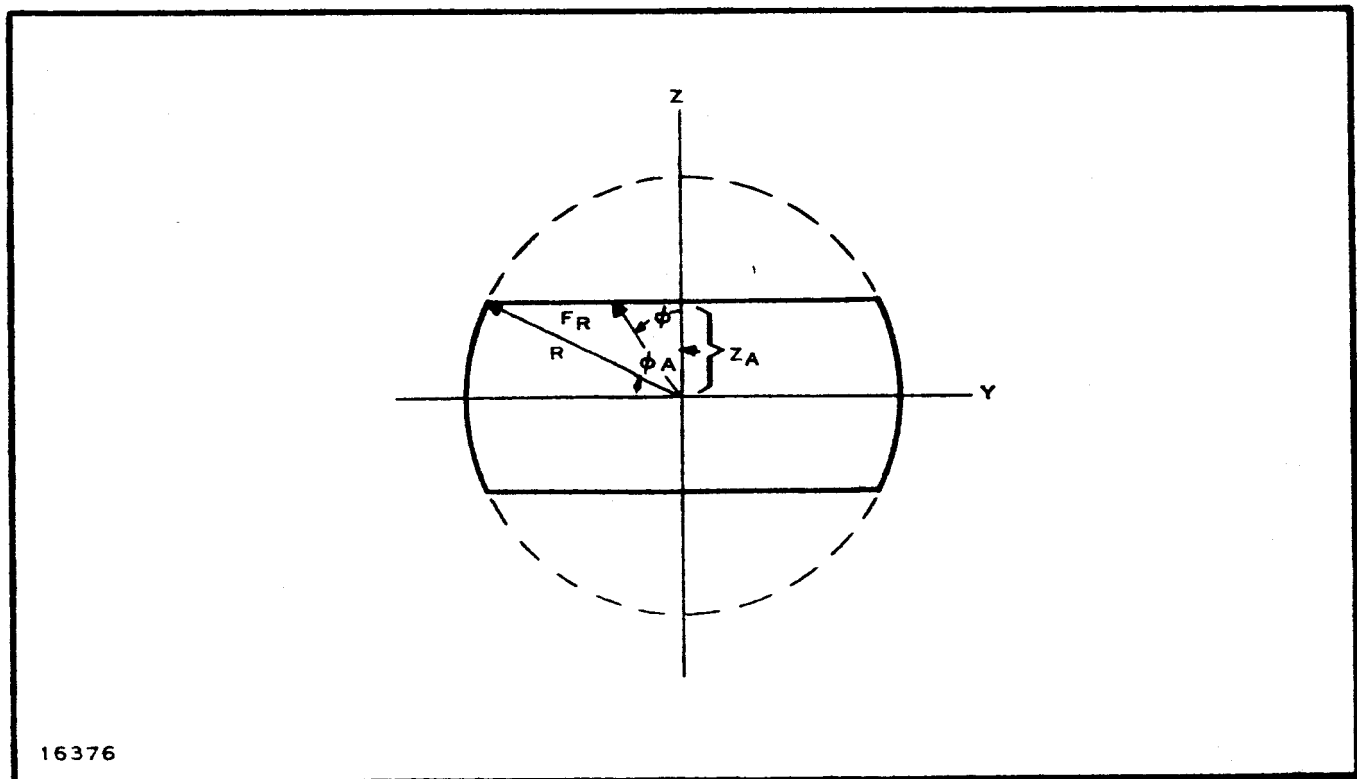


Figure 12. A Cross Section of the Truncated Sphere

$$p(Z) = \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{Z^2}{2}\right)} \quad (56)$$

where the variance is normalized to unity. Then,

$$H = Z_0 \sqrt{V_{x,N}} \quad (57)$$

and

$$V_{x,N} = \left(\frac{H}{Z_0}\right)^2. \quad (58)$$

The parameters of the probability density functions of the individual random variables, such as M , the maximum allowable value of magnitude of the dipole moment, may then be adjusted so that Equation (58) is satisfied.

J. Other Physical Configurations

All computations to this point considered a spherical electronics package. Other configurations may be introduced by alterations of the joint probability density function, $p(m, r, \phi_1, \phi_2, \theta_1, \theta_2)$. A sphere, truncated by two planes parallel to the X - Y plane and passing through the points Z_A and Z_B as shown in Figure 11, would produce a configuration similar to the octagonal-sided electronics package of the Mariner C. Figure 12 shows a cross-section of the truncated sphere. Consider a dipole whose position is uniformly distributed within the confines of the package. If the position of the dipole is described by the random variables r , ϕ_1 , and θ_1 , then ϕ_1 can be considered as being uniformly distributed between 0 and π ; θ_1 can be uniformly distributed between 0 and 2π ; but the range of r is dependent upon ϕ_1 . Let the maximum value of the vector r be designated as F_r . Then,

$$F_r = \begin{cases} \frac{Z_A}{\cos \phi} & \text{for } 0 \leq \phi_1 \leq \phi_A \\ & \text{and } \pi - \phi_A \leq \phi_1 \leq \pi \\ R & \text{for } \phi_A \leq \phi_1 \leq \pi - \phi_A \end{cases} \quad (59)$$

The joint probability density function of ϕ_1 and r by the theorem of compound probability must be written as

$$p(\phi_1, r) = p(\phi_1) p(r | \phi_1) \quad (60)$$

where $p(\phi_1)$ is the unconditional probability density function of ϕ_1 and $p(r | \phi_1)$ is the conditional density function of r given that a value of ϕ_1 has occurred.

Then

$$p(\phi_1, r) = \begin{cases} \frac{\cos \phi}{\phi_A Z_A} & \text{for } 0 \leq \phi_1 \leq \phi_A \\ & \text{and } \pi - \phi_A \leq \phi_1 \leq \pi \\ \frac{1}{\phi_A R} & \text{for } \phi_A \leq \phi_1 \leq \pi - \phi_A \\ 0 & \text{elsewhere} \end{cases} \quad (61)$$

Where ϕ_1 and r are independent of the other random variables, the joint probability density function of all random variables equals $p(\phi_1, r) p(m, \phi_2, \theta_1, \theta_2)$. When integrating this joint probability density function [in the manner of Equation (27)], the integral over the range of r should be evaluated first since the limit of this range involves the variable ϕ_1 . This result, $p(H_x)$, may then be handled in a manner analogous to the spherical case previously investigated to yield similar results.

K. Probability of Magnetic Stability

The concept of a statistical model may also be used under more restrictive conditions. For example, consider a unit on the spacecraft which is magnetic but must be treated separately from the general statistical model of the electronics box. Such a unit might be a transformer which has a very large magnetic moment or some piece of hardware in the near vicinity of the sensor.

As a particular case, assume that parameters such as position and orientation of an equivalent dipole can be determined and considered constant. Assume that the magnitude of the magnetic moment decays with time, but decays in a manner which is not deterministic. For example, the magnitude of the moment might be described by the equation

$$m = M e^{-bt} \quad (62)$$

where b is described by the probability density function, $p(b)$. A component of the magnetic field intensity, such as H_x , can be expressed by the relation

$$H_x = \frac{3m}{2r^3} \sin 2\phi \cos \theta; \quad (63)$$

but all parameters are constant except m , so,

$$H_x = KM e^{-bt} \quad (64)$$

The problem is then to determine a probability density function for H_x . H_x is a single-valued, continuous function of b ; therefore,

$$p(H_x) = p(b) |J| \quad (65)$$

Solving for b in terms of H_x ,

$$b = g(H_x) = -\frac{\ln}{t} \left(\frac{H_x}{KM} \right) \quad (66)$$

$$J = \frac{\partial g(H_x)}{\partial H_x} = -\frac{1}{tH_x} \quad (67)$$

Therefore,

$$p(H_x) = \frac{1}{tH_x} p[b = g(H_x)] \quad (68)$$

This type solution could also apply to the case where some probability density function is derived or estimated considering the magnetization stability and expected environments.

L. References

¹ Wilbur B. Davenport, Jr. and William L. Root, Random Signals and Noise (New York: McGraw-Hill Book Company Inc., 1953).

² Y. W. Lee, Statistical Theory of Communications (New York: John Wiley and Sons, 1960).

³ Karl Pearson, Tables for Statisticians and Biometricians, Part I (London: Cambridge University Press, 1924).

⁴ Alfred K. Susskind, Notes on Analog-Digital Conversion Techniques (New York, Technology Press and John Wiley and Sons, 1957).

⁵ Ivan S. Sokolnikoff and Elizabeth S. Sokolnikoff, Higher Mathematics for Engineers and Physicists (New York: McGraw-Hill Book Company Inc., 1941).

IV. CONCLUSIONS AND RECOMMENDATIONS

One of the major problems in developing a nonmagnetic component, system, or spacecraft is personnel education—acquainting those responsible with the considerations involved. The handbook of terms, formulas, units and measuring methods written as part of this contract should be of great aid to those working in the area of nonmagnetic components and circuits. An attempt was made to explain or eliminate ambiguities in definitions and unit system usage and to explain the origin of some of the magnetic effects the designer must consider. Therefore, it is recommended that this handbook be given wide distribution by JPL.

A component test method was developed that is accurate, repeatable, sensitive, and relatively simple. It is readily adaptable for use by manufacturers of nonmagnetic hardware as well as in a comprehensive component test program leading to the development of a preferred parts list of nonmagnetic components. It is recommended that the list of components selected for testing not be severely limited in an attempt to limit the size of the resulting preferred parts list. Since there are so few known nonmagnetic parts available, the results of the recommended test program will probably cause the size of the preferred parts list to be self-limiting.

It should be noted that the derivation of a stochastic model for the magnetic field of a spacecraft is just the first step in a complete analysis of the problem. The present study has shown that the problem can be handled mathematically and that it can be reduced to a readily usable form. The success or practicality of this approach will depend on the inputs to the computation and on the definition of the probability density functions. After component data has been obtained in a measurement program, additional study will be required to determine probability density functions of the random variables and incorporate the statistics in a stochastic model. This could lead to the solution of the spacecraft magnetic field problem in a generalized plug-in format.

In December 1963, Texas Instruments was asked by JPL to comment on several approximate methods of combining the effects of a number of dipole moments. In general, the validity of these approximations is highly questionable; we can neither substantiate nor disprove them. Since this is the central problem being studied in statistical modeling, it is not apparent what approximations may be made until the statistical studies have been carried beyond this preliminary stage. The assumptions required for a given approximation may, through experience, be found to be usable for a particular application. No statement can be made on their validity in a general situation, particularly for cases involving either clustering or preferred orientation of dipoles.

The areas on which additional effort should be concentrated are the measurement of electronic component magnetic field and a study of magnetization stability.



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APPENDIX A
COMPONENTS FOR MAGNETIC
EVALUATION

APPENDIX A

COMPONENTS FOR MAGNETIC EVALUATION

I. MAGNETIC EVALUATION PARTS LIST

A. Contents

This parts list is divided into two sections. Table I is a list of parts which, through Texas Instruments magnetometer development programs, have already been tested and on which past history indicates that the parts are probably nonmagnetic. Zero test quantities have been called out on this list; therefore, it is included primarily for information purposes. Table II is a suggested list of parts for magnetic evaluation. This list is based on JPL's Preferred Parts List and on identifiable parts from the Mariner C component parts surveys.

B. Format Explanation

The first column on the parts list is a card sequence number for sorting and identification purposes. The second and third columns contain the manufacturers Federal Stock Codes and part number of the component to be tested. Manufacturer names and addresses are keyed to the Federal Stock Codes in Table IV.

Columns 4 through 7 provide a complete description of the component by general class, specific type, and where available, information on package and lead styles and materials.

Columns 8 and 9 contain the total quantity of components to be tested initially and the number of different production lots from which the total is to be sampled. A "T" in these columns indicates special devices requiring 100-percent testing.

Column 10 contains information on the known magnetic properties of the component. An "N" appears in this column for components whose past history indicates they are nonmagnetic. An "M" indicates that the component is known to contain magnetic materials.

Columns 11 and 12 refer to notations in the JPL Preferred Parts List. An "S" in column 11 and an "H" in column 12 indicate respectively that the component is a stock item and a high-reliability part.

The last column contains the card sequence numbers of remarks which pertain to the given component and which, in some cases, also give control requirements. These remarks are listed in numerical order by card sequence number in Table III, Remarks. An "NC" in this column indicates that, to the best of our knowledge, this item is not used in the Mariner C spacecraft electronics.

Table I. Nonmagnetic Parts List

SEQ	MFG CODE	MFG PART NUMBER	COMPNT CLASS	COMPNT TYPE	PACKAGE	LEADS	SH			REMARKS
							TQ L	TI	EU O MOR	
							SA T	ACE	TN S GKL	
003	91418	TYPES B,U	CAP	CER	DISC STD	TND CU	00	0	N	
017	84171	TYPE DM	CAP	MICA	EPOX MLD	TND CU,X	00	0	N	810
020	04099	SERIES MMW	CAP	MYLAR	TUB MIN	TND CU	00	0	N	NC
021	89037	TYPE 602	CAP	MYLAR	TUB	X	00	0	N	NC 812
025	13934	TYPES 2,3	CAP	MYLAR			00	0	N	NC
026	99120	TYPE LT1	CAP	MYLAR			00	0	N	NC 814
031	72354	TYPE 182T	CAP	P MIL (K)	TUB STD	AU/CO	00	0	NSH	815
038	89037	PT4-XXXX	CAP	PLASTIC	TUB,FLT	X	00	0	N	NC 816
043	05079	TYPE TS	CAP	TA DRY	UMIN		00	0	N	NC 803
044	08798	29F600 + UP	CAP	TA DRY	TUB STD	X	00	0	XS	819
057	71590		CAP	TRM,CER	MIN		00	0	N	NC
059	95712	10239-1	CONN	COAX			00	0	N	NC
061	95077	TYPE 8166B	CONN	COAX,TNC	AG/RH/AL	AG/BE CU	00	0	N	NC 857
063	91662	5101.0613CS	CONN	PCB			00	0	N	NC 859
064	71468	DAM-DEM,NM10	CONN	RECT	S MIN		00	0	NSH	
065	71785	DAM-DEM,NM10	CONN	RECT	S MIN		00	0	NSH	NC
067	77820	TYPE JC	CONN	RD	MIN MLTI		00	0	N	
069	71468	DB-25P	CONN				0	0	N	NC
075	12060	SER DI-52	DIODE	PWR	CER MIN	AG	00	0	N	NC 824
081	14099	SC2	DIODE	PWR	CER	AG	00	0	N	NC 828
085	01295	1N2069	DIODE	PWR	EPXY	AG	00	0	N	NC 831
096	12060	SER 1D11B	DIODE	REF GP	CER MIN	AG	00	0	N	NC 834
116	12060	SER DI645	DIODE	SIG/CMPT	CER MIN	AG	00	0	N	NC
127	01295	B-304	DIODE	SIG/CMPT	UDIODE	PT	00	0	N	855
154	01121	TYPES CB,EB	RES FXD	C COMP	PLAS MLD	TND CU	00	0	NS	
157	12401	TYPE GBT	RES FXD	C COMP	PLAS MLD	X	00	0	N	900
171	77764	TYPE LAF	RES FXD	PREC WW			00	0	N	NC 910
173	07088	EP20	RES FXD	WWPREC ENC	EPXY	TIN/BR	00	0	N	913
187	91637	SER 600NM	RES VAR	TRM XX	EPXY	ENAM,IRE	00	0	N	NC 919
188	X	MODEL 101	RES VAR	TRM WW	PLAS	TND CU	00	0	N	NC 920
193	91929	21SX1-T	SWITCH	SNP ACTN		TERM	00	0	N	NC
199	98291	FT-SM-8P30	TERM	FTRU STOFF	TEFLON		00	0	N	923
201	08795	DF-101	TERM	SOLDER SLV	IPO		00	0	N	924
204	04867	TS-111-6M	TERM	STOFF	TEFLON	X	00	0	N	927 928
205	04867	622G	TERM	STOFF	TEFLON	X	00	0	N	927 929
208	13088	92-1500-X	TERM	STOFF		AU/BE CU	00	0	N	
209	08145	1485A,1486A	TERM	STOFF			00	0	N	930
210	04867	605	TERM	TEST POINT			00	0	N	927
211	98291	SKT 8-P20	TERM	TEST POINT	TEFLON		00	0	N	931
214	73168	KA31L1	TMTR	DISC	MIN	TND CU	00	0	N	NC
237	01281	PT629	TSTR	PWR	SPL		00	0	N	NC
250	01281	PT618	TSTR	PWR,VHF	SPL	SPL	00	0	N	NC
251	01281	PT1574A	TSTR	PWR,VHF	SPL	SPL	00	0	N	956
265	05973	34967	W + CBL	COAX JKT		INS	00	0	N	
266	07497	RG-58C/U	W + CBL	COAX JKT		INS	00	0	N	
271	15116	50-3946	W + CBL	COAX JKT		INS	00	0	N	966
275	X		W + CBL	COAX JKT		INS	00	0	N	968 969
281	05973	SUPEREX-E	W + CBL	HOOKUP SHLD		INS	00	0	N	
282	12515	X	W + CBL	HOOKUP SHLD		INS,X	00	0	N	971 972
283	07497	RG-108/U	W + CBL	SHLD TWISTED PR		INS	00	0	N	
284	06090	62-202	W + CBL	SHLD TWISTED PR		INS,IPO	00	0	N	973
285	18626	=28 NICHROME	W + CBL	RESISTANCE			00	0	N	

Table II. Magnetic Evaluation Parts List

SEQ	MFG CODE	MFG PART NUMBER	COMPNT CLASS	COMPNT TYPE	PACKAGE	LEADS	TQ L	SH TI	EU O MOR	SA T ACE	IN S GKL	REMARKS
001	71590	TYPE DD	CAP	CER	DISC STD		10 2	S				801
002	56280	TYPE 10TS	CAP	CER	DISC SID	IND CU	10 2					802
004	00656	HMC80,HMC81	CAP	CER LV	TUB SMIN	TND CU	10 2	SH				
005	15450	TYPE 386	CAP	CER LV	TUB MIN	TND CU	10 2					
006	15287	TYPE SCD	CAP	CER LV	TUB SMIN		10 2					803
007	95275	TYPE CK	CAP	CER LV	RECT STD	X	10 2	X				804 805
008	96733	TYPE CK	CAP	CER LV	RECT MIN	TND CU	10 2					
009	96733	TYPE RH06	CAP	CER LV	RECTSMIN	IND CU	10 2					
010	00650	TYPE BLF	CAP	CER	FTRU STD		10 2					NC 806
011	91984	TYPE 287	CAP	CER	FIRU SID		10 2					
012	00656	TYPE BLS	CAP	CER	STOFFSTD		10 2	S				807
013	91984	TYPE 293	CAP	CER	STOFFSTD		10 2					NC
014	14674	CYFR,LVL B	CAP	GL/PORC	RECT STD	CU/Ni-Fe	10 2	MSH				
015	95275	TYPE CY	CAP	GL/PORC	RECT STD	X	10 2	XX				805 808
016	84171	TYPE CM	CAP	MICA	RECT STD		10 2	S				809
018	76433	TYPE CM	CAP	MICA	RECT SID		10 2					NC
019	76433	TYPE DM	CAP	MICA	RECT SID		10 2					NC
021	99515	TYPE EW-150	CAP	MYLAR	TUB MIN	CU/STEEL	10 2	M				NC
022	89037	TYPE 617G	CAP	MYLAR	TUB SID		10 2					813
023	89037	TYPE 627G	CAP	MYLAR	TUB MIN		10 2					
024	89037	TYPE 683G	CAP	MYLAR	TUB MIN		10 2					
027	99515	TYPE DE	CAP	MYLAR,MET	TUB MIN		10 2					803
028	99515	TYPE EG	CAP	MYLAR,MET	RECT MIN	CU/STEEL	10 2	M				
029	56289	TYPE 188F	CAP	P	TUB STD		10 2					
030	56289	TYPE 103P	CAP	P,FTRU	TUB FTRU		10 2					
032	56289	TYPE 195P	CAP	P MIL	TUB STD		10 2					NC
033	00656	TYPE P323ZN	CAP	P MET	TUB MIN		10 2	S				
034	56289	TYPE 118P	CAP	P MET	TUB MIN		10 2					
035	56289	TYPE 121P	CAP	P MET	TUB MIN		10 2					
036	96733	TYPE 48	CAP	P MET	TUB MIN		10 2					NC
037	99515	TYPE PSG	CAP	PLASTIC	RECT MIN	CU/STEEL	10 2	M				
039	08798	TYPE 7K(MM)	CAP	TA DRY	TUB SMIN		10 2	SH				
040	05397	TYPE J	CAP	TA DRY	TUB SMIN	CUWELD	10 2	MSH				
041	56289	TYPE 150D	CAP	TA DRY	TUB SMIN	TND NI	10 2	M				
042	56289	TYPE 350D(MM)	CAP	TA DRY	TUB SMIN	TND NI	10 2	MSH				810
045	56289	TYPE 120D	CAP	TA WT FOIL	TUB STD		10 2	X				820
046	08798	TYPE 5K(MM)	CAP	TA WT FOIL	TUB STD		10 2	SH				NC
047	56289	TYPE 109D	CAP	TA WT SLUG	TUB SMIN		10 2					
048	56289	TYPE 130D	CAP	TA WT SLUG	TUB MIN		10 2					
049	21520	TYPE HP	CAP	TA WT SLUG	BELL MII		10 2	S				NC
050	99127	TYPE S HT,LV	CAP	TEFLON	TUB SMIN	TND CU	10 2					
051	12517	TYPE TA	CAP	TEFLON	TUB MIN	TND CU	10 2					
052	72928	TYPE XT	CAP	TEFLON	TUB STD		10 2					NC
053	99217	TYPE TC	CAP	TEFLON	TUB STD		10 2					NC
054	96733	TYPE S 97,98	CAP	TEFLON	TUB STD	TND CU	10 2					
055	74970	JMC 2954	CAP	TRM,AIR	MIN		10 2					
056	93738	TYPE AP39	CAP	TRM,AIR	MIN	AG/BRASS	10 2					
058	73899	TYPE VC21G	CAP	TRM,GL	CYLIN,GL	X	10 2	M				822

Table II. Magnetic Evaluation Parts List (Continued)

							SH				
							TQ	L	TI		
							EU	O	MOR		
							SA	T	ACE		
SEQ	MFG CODE	MFG PART NUMBER	COMPNT CLASS	COMPNT TYPE	PACKAGE	LEADS	TN	S	GKL	REMARKS	
060	15116	TYPE 31-50	CONN	COAX SMIN	X	AU/AG	10	1		856	
062	95077	X	CONN	COAX,TNC	AG/BRS	AU/BE	CU	10	1	858	
066	77820	TYPE SC	CONN	RD,MLTIPIN	AL SHELL	AU/BE	CU	10	1	860	
068	77820	TYPE JB	CONN	RD Q DISC	MIN			10	1	SH NC	
070	09214	3N60	SCR		TO-12	KOVAR	20	4	M	823	
071	09214	TYPE C5D	SCR	LO CUR	TO-5	KOVAR	20	4	M	823	
072	09214	C35 SER	SCR	PWR	TO-48	STDMT	20	4			
073	96341	1N831AM	DIODE	UWAVE MXR	GL SMIN		20	4			
074	96341	MA450C	DIODE	UWAVE VRGR	PLUGIN		20	4			
076	72699	1N248A	DIODE	PWR	DO-5		20	4	S	NC	
077	72699	1N1200	DIODE	PWR	STDMT		20	4		NC	
078	72699	1N1583	DIODE	PWR	DO-4		20	4	X	NC 825	
079	04713	1N2611	DIODE	X	FLNGLESS		20	4	X	826	
080	01281	1N645	DIODE	X	GL SMIN		20	4	XX	827	
082	14099	SCBR15A	DIODE	PWR	SPL	TERM	20	4			
083	01295	G681EHR	DIODE	PWR	GL SMIN		20	4	X	829	
084	01295	1N1124	DIODE	PWR	DO-4	STDMT	20	4	S	830	
086	12065	1N248A	DIODE	PWR	DO-5		20	4	S	NC	
087	12065	1N645	DIODE	PWR	GL SMIN		20	4		832	
088	12065	1N1184	DIODE	PWR	STDMT		20	4			
089	12065	1N1200	DIODE	PWR	STDMT		20	4		NC	
090	12065	1N1583	DIODE	X	DO-4		20	4		833	
091	07263	FSP12	DIODE	PWR,BR	TO-18	KOVAR(4)	20	4	M	823	
092	07263	FSP-36	DIODE	PWR,BR	TO-12	KOVAR	20	4	M	823	
093	07263	FSP453	DIODE	PWR,BR			20	4			
094	01281	PS2414	DIODE	PWR,BR	SPL		20	4			
095	07263	FA4049	DIODE	PWR,QUAD	EPXY	DUMET(8)	20	4	M	823	
097	99942	1N1815	DIODE	REF GP	STDMT		20	4		NC	
098	99942	1N2810A	DIODE	REF GP	DIAMOND		20	4		NC 835	
099	99942	PG971B	DIODE	REF GP			20	4		836	
100	99942	R1603	DIODE	REF GP	X	NI	20	4	M	837	
101	04713	1N1351	DIODE	REF GP	DO-4	STDMT	20	4		838	
102	04713	1N1807	DIODE	REF GP	STDMT		20	4		NC	
103	04713	1N2810A	DIODE	REF GP	DIAMOND		20	4		NC 839	
104	04713	1N3283	DIODE	REF GP	DO-7		20	4		840	
105	01281	1N459A	DIODE	X	GL SMIN		20	4	XX	841 842	
106	01281	PS1203B	DIODE	REF GP			20	4			
107	01295	1N746A	DIODE	X	GL SMIN		20	4	XX	841	
108	12065	1N761	DIODE	REF GP	DO-7		20	4		843	
109	12065	1N1805	DIODE	REF GP	STDMT		20	4		NC 844	
110	12065	1N2033	DIODE	REF GP			20	4		845	
111	12065	1N3029	DIODE	REF GP	GL SMIN		20	4		846	
112	07910	CD4246	DIODE	REF PREC	GL SMIN		20	4		847	
113	99942	1N821	DIODE	REF PREC	DO-7		20	4	X	848	

Table II. Magnetic Evaluation Parts List (Continued)

SEQ	MFG CODE	MFG PART NUMBER	COMPNT CLASS	COMPNT TYPE	PACKAGE	LEADS	SH			REMARKS
							TQ L	TI	MOR	
							SA T	ACE		
							TN S	GKL		
114	04713	1N821	DIODE	REF PREC	DO-7		20 4			NC
115	X	AM602	DIODE	SIG/CMPTR	DO-7		20 4			849
117	07263	1N459A	DIODE	SIG/CMPTR	GL SMIN	DUMET	20 4	X		823 850
118	07263	1N916	DIODE	SIG/CMPTR	DO-7	DUMET	20 4	MS		823
119	07263	F0300	DIODE	SIG/CMPTR	GL SMIN	DUMET	20 4	MAX		823 851
120	73293	HR1B	DIODE	X	GL SMIN		20 4			852
121	14552	MC1291	DIODE	SIG/CMPTR	UDIODE		20 4	XX		853
122	94145	1N3730	DIODE	SIG/CMPTR	GL SMIN		20 4			
123	07713	1N692	DIODE	SIG/CMPTR	DO-7		20 4			
124	01295	1N916A	DIODE	SIG/CMPTR	DO-7		20 4	S		
125	01295	1N3579	DIODE	SIG/CMPTR	GL MIN		20 4			
126	01295	TI256	DIODE	SIG/CMPTR	UDIODE		20 4			854
128	93332	D4242F	DIODE	VARACTOR	GL		20 4			
129	11313	TYPE 11854-2	IND	CHOKE			10 2			
130	99800	SER 1537	IND	CHOKE,RF	MLD	T CUWELD	10 2	M		823
131	99800	SER 1840	IND	CHOKE,RF	MLD	T CUWELD	10 2	M		823
132	99800	SER 2500	IND	CHOKE,RF	MLD	T CUWELD	10 2	M		823
133	72259	WEE-DUCTOR	IND	CHOKE,RF	EPXY,MLD		10 2			
134	09349	SPECIAL	IND	LO FREQ			10 2			863
135	80223	TYPE ML	IND	LO FREQ	SMIN	NI/DUMET	10 2	M		823 864
136	80223	TYPE MM	IND	LO FREQ	SMIN		10 2			865
137	81095	SP 106, 108	IND				10 2			
138	82110	ST18-DAC96	LCNG CD	CL1,2,3			10 FT			
139	07263	MLG(GATE)	UCTT	ULOGIC SER	TO-5		20 4			NC 866
140	01295	SN510(FF)	UCTT	SER51 UCKT	SPL GL KOVAR		20 4	M		NC 823 867
141	94875	MARK 1	RELAY	MIN			10 1			NC
142	45402	MARK 11	RELAY	MIN			10 1			NC
143	88997	UN329734-017	RELAY	MIN			10 1			NC
144	09026	TYPE BR-7X	RELAY	PWR			10 1			
145	71482	TYPE F	RELAY	SMIN	XTAL CAN		10 1			
146	99699	SER J	RELAY	SMIN			10 1			NC
147	00614	9224-5691	RELAY	PWR			10 1			NC
148	00614	9227-4972	RELAY	PWR			10 1			NC
149	77342	TYPE SL	RELAY	SMIN	XTAL CAN		10 1	M		869
150	78277	TYPE 32	RELAY	SMIN	HERM SLD		10 1	M H		869
151	78277	TYPE 33	RELAY	SMIN	HERM SLD		10 1	M		869
152	08798	TYPE 3SAF	RELAY	UMIN	XTAL CAN		10 1			
153	03034	AAD-7100N	RELAY				10 1			

Table II. Magnetic Evaluation Parts List (Continued)

SEQ	CODE	MFG PART NUMBER	COMPNT CLASS	COMPNT TYPE	PACKAGE	LEADS	- SH			REMARKS
							TO L	TI	EU O MOR	
							SA T	ACE	TN S GKL	
155	01121	TYPE GB	RES FXD	C COMP	PLAS MLD		10	2	S	
156	01121	TYPE TR	RES FXD	C COMP	PLAS MLD		10	2		
158	07716	TYPE CE	RES FXD	MET FLM	EPXY MLD	X	10	2		901
159	01295	TYPE CG	RES FXD	GL C FLM	GL	TND CU	10	2	MSH	902
160	12401	TYPE XLT	RES FXD	GL MET FLM	GL	AU/DUMET	10	2	M H	NC 823
161	63060	TYPE RX-1	RES FXD	HI-MEG	GL		10	2		
162	01295	TYPE CDM	RES FXD	C FLM	EPXY MLD	TND CU	10	2	S	903
163	09132	TYPE 985	RES FXD	MET FLM	EPXY MLD	TND CU	10	2	M	NC 904
164	07716	EM, ME(T-O)	RES FXD	MET FLM	PLAS MLD	X	10	2		905
165	01295	TYPE CD 1/8	RES FXD	PREC C FLM	UMIN, PNT	TND CU	10	2		
166	91637	TYPE RH	RES FXD	WW PWRPREC	SILICONE		10	2	S	906
167	91637	TYPE G	RES FXD	WW PWRPREC	SILICONE	AU/CUWLD	10	2	M	
168	91637	TYPE RS	RES FXD	WW PWRPREC	SILICONE	TND CU	10	2	S	907
169	07180	TYPE M()W	RES FXD	WW PWRPREC	ANDZD AL		10	2		NC 908
170	02985	TYPE TS	RES FXD	WW PWRPREC			10	2		NC 909
172	07150	X	RES FXD	WWPREC ENC	EPXY MLD		10	2		NC 911
174	01295	TYPE TC	RES FXD	SENSISTOR	TO-5	KUVAR	10	2	M	823
175	01295	TYPE TGX-01	RES FXD	SENSISTOR	GL, UMIN	RODAR	10	2		
176	01295	TYPE TM	RES FXD	SENSISTOR	EPXY		10	2		
177	01121	TYPES L, K	RES VAR	GP C COMP		TERM	10	2		NC 914
178	97979	TYPES D, R	RES VAR	GP C COMP		TERM	10	2		NC
179	98659	ALL TYPES	RES VAR	PREC C FLM		TERM	10	2		NC
180	80740	TYPE 6203	RES VAR	PREC WW		TERM	10	2		915
181	80294	TYPE 3000	RES VAR	PREC WW	PLAS	X	10	2		916
182	04454	ALL TYPES	RES VAR	PREC WW			10	2		NC
183	02111	ALL TYPES	RES VAR	PREC WW		TERM	10	2		NC
184	80294	TYPE 3051	RES VAR	TRM C COMP		X	10	2		NC 918
185	80294	TYPES 224, 220	RES VAR	TRM WW	STLS STL	X	10	2		918
186	80294	TYPE 3250	RES VAR	TRM WW	PLAS	X	10	2		918
189	12617	TYPE MSRG-15	SWITCH	DRY REED	GL		10	1		921
190	91929	TYPE 1HM1	SWITCH	LIM PREC	HERM SLD	TERM	10	1		
191	82647	AT-1, BASIC	SWITCH	LIM PREC	HERM SLD		10	1		NC
192	91929	TYPE V3-245	SWITCH	SENSTV	HI-TEMP	SCREW	10	1		
194	04426	TYPE 16	SWITCH	SNP ACTN	S MIN	TERM	10	1		NC
195	14604	TYPE 3100-1	SWITCH	THERMAL			10	1		
196	05791	TYPE 2426-125	TERM	FTRU STOFF	GNOSTOFF	AG/GRS	20	4		
197	15116	TYPE 4532-A	TERM	FTRU STOFF		AU/GRS	20	4		
198	15116	TYPE 1490-A9	TERM	FTRU STOFF	TEFLON	AU/GRS	20	4		
200	98291	FT1550DTUR	TERM	FTRU STOFF	TEFLON	AU/GRS	20	4		
202	05791	3600 SERIES	TERM	SPLIT TYPE		AG/GRS	20	4		925

Table II. Magnetic Evaluation Parts List (Continued)

SEQ	MFG CODE	MFG PART NUMBER	COMPNT CLASS	COMPNT TYPE	PACKAGE	LEADS	SH			REMARKS
							TQ L	TI	EU O MOR	
							SA T	ACE	TN S GKL	
203	15116	4500,5000SER	TERM	SPLIT TYPE		AU/BRS	20	4		926
206	05791	TYPE S-1-T	TERM	STOFF	TEFLON	AG/BRS	20	4		
207	98291	TYPE ST250SL	TERM	STOFF	TEFLON		20	4		
212	15801	TYPE GA51L3	TMTR	BLAD	GL/SMIN	PT/IR	10	2		
213	15801	TYPE JA41L2	TMTR	DISC	MIN	TND CU	10	2		
215	83186	TYPE 33A5	TMTR	PROBE	GL/MIN	T/DUMET	10	2	M	823
216	15801	TYPE QB41J1	TMTR	RDD	STD	TND CU	10	2		932
217	93410	TYPE MX-1	THERMO	DISC	SMIN		10	2		NC
218	82647	TYPE C4344,M1	THERMO	DISC	MIN		10	2		NC
219	73168	TYPE 32400	THERMO	RECT	MIN		10	2		NC
220	93929	TYPE C8,C8-52	THERMO	TUB ADJ	MIN		10	2		NC
221	01961	TYPE 2230	XFMR	BLKG OSC	EPXY	TND CU	10	1		934
222	80223	TYPE H68	XFMR	BLKG OSC	MIN		10	1		
223	81095	SP11	XFMR	INTRSTG	EPXY	NI	10	1	M	935
224	09349		XFMR	SPL TYPES			T	T		
225	80223	TYPE DI-T	XFMR	VARIOUS	SMIN	TND CU	10	1		
226	80223	TYPE DU-T	XFMR	VARIOUS	MIN	TND CU	10	1		936
227	12040	TYPE NS-3001	TSTR	CHOPPER	TO-18	KOVAR(4)	20	4	M	823 937
228	07713	TYPE 2N943	TSTR	X	TO-18	KOVAR	20	4	MS	823 938
229	01295	SM2704	TSTR	CHOPPER	TO-18	KOVAR(4)	20	4	M	823 939
230	07263	2N2060	TSTR	DUAL TSTR	TO-5	KOVAR(6)	20	4	M	823 940
231	01295	2N2642	TSTR	DUAL TSTR	TO-5	KOVAR(6)	20	4	M	823
232	00872	FE200	TSTR	FLT	TO-5	KOVAR	20	4	MS	NC 823
233	07263	2N708	TSTR	MED PWR	TO-18	KOVAR	20	4	MXX	823 941-43
234	07263	2N1131	TSTR	MED PWR	TO-5	KOVAR	20	4	MXX	823,941,4,5
235	04713	2N2222	TSTR	MED PWR	TO-18	KOVAR	20	4	MXX	823 946
236	01295	2N1613	TSTR	X	TO-5	KOVAR	20	4	MXX	823 947-48
238	02735	2N1490	TSTR	PWR	DMND SPL		20	4		NC
239	07256	2N1016	TSTR	PWR	SPL	STDMT	20	4	X	NC 949
240	07256	2N1490	TSTR	PWR	DMND SPL		20	4		NC
241	07256	2N2034	TSTR	PWR	TO-5	KOVAR	20	4		823
242	07256	2N2035	TSTR	PWR	TO-8	KOVAR	20	4	M	823 950
243	01295	2N389	TSTR	PWR	SPL,RECT		20	4		NC 951
244	01295	2N1048B	TSTR	PWR	SPL	STDMT	20	4		952
245	01295	2N1724	TSTR	PWR	SPL	STDMT	20	4		
246	01295	2N2150	TSTR	PWR	SPL	STDMT	20	4		
247	01295	SP977	TSTR	PWR	SPL STDMT	KOVAR	20	4	M	823 953
248	12065	2N389	TSTR	PWR	SPL RECT		20	4	X	NC 954
249	05277	2N1016	TSTR	PWR	SPL STUD		20	4	X	955
252	73293	2N1234	TSTR	SH SIG	TO-5	KOVAR	20	4	M	823 957

Table II. Magnetic Evaluation Parts List (Continued)

SEQ	MFG CODE	MFG PART NUMBER	COMPNT CLASS	COMPNT TYPE	PACKAGE	LEADS	SH			REMARKS
							TG	L	TI	
							EU	O	MOR	
							SA	T	ACE	
							TN	S	GKL	
253	12040	2N328A	TSTR	SM SIG	TO-5	KOVAR	20	4	MX	NC 823 958
254	11530	2N859	TSTR	SM SIG	TO-18	KOVAR	20	4	M	823 959
255	94145	2N1656	TSTR	SM SIG	TO-5	KOVAR	20	4	M	823
256	07713	2N328A	TSTR	SM SIG	TO-5	KOVAR	20	4	MX	823 960
257	01295	2N930	TSTR	X	TO-18	KOVAR	20	4	MX	823 961
258	RESERVED									
259	09213	2N338	TSTR	SWITCH	TO-5	KOVAR	20	4	M	823 962
260	08732	2N886	TSTR	TRIGISTOR	TO-18	KOVAR	20	4	M	963
261	08732	3C60A	TSTR	X	TO-9	KOVAR	20	4	M	823 964
262	09213	2N491B	TSTR	UJT	TO-5	KOVAR	20	4	MS	823 965
263	07263	2N917	TSTR	VHF AMP	TO-18	KOVAR(4)	20	4	M	823
264	81453	TYPE 5886	TUBE EL	ELECTROM	SMIN		10	1		NC
267	07497	RG-214/U	W + CBL	COAX	JKT-PVC	INS-POLYETH	10FT		S	
268	03890	RG-11A/U	W + CBL	COAX	JKT-PVC	INS-POLYETH	10FT		S	
269	15116	TYPE 50-3804	W + CBL	COAX	JKT-PVC	INS-POLYETH	10FT		S	
270	15116	50-3920CW	W + CBL	COAX	JKT-TEFLON	INS-TEFLON	10FT			
272	82879	RG-213/U	W + CBL	COAX	JKT-PVC	INS-POLYETH	10FT		S	
273	07145	RG/U TYPES	W + CBL	COAX	JKT-PVC	INS-POLYETH	10FT		S	967
274	07145	RG-141A/U	W + CBL	COAX	JKT-TEFLON	INS-TEFLON	10FT			
276	X	TYPE E	W + CBL	HOOKUP		INS-	10FT			970
277	99114	TYPE E	W + CBL	HOOKUP		INS-TEFLON	10FT			
278	90484	TYPE E	W + CBL	HOOKUP		INS-TEFLON	10 FT		S	
279	12515	TYPE E	W + CBL	HOOKUP		INS-TEFLON	10FT			
280	98053	TYPE E	W + CBL	HOOKUP		INS-TEFLON	10FT			
286	71400	TYPE GFA-1/20 FUSE			SMIN	TND CU	10	1		
287	03911	TYPE CL705L	PHOTOCELL	CAD S	TO-5		20	4		
288	08806	TYPE NE-23	LAMP	NEON	TUB	CU	10	1		
289	X	MC-180	CORE	MAGNETIC	TOROID		10	2	M	974

Table III. Remarks

801 INCLUDES TYPE TC-N47C
 802 TEMPERATURE STABLE
 803 MOLDED EPOXY PACKAGE
 804 INCLUDES TYPE CK(S) AND TYPE VK
 805 AVAIL LEADS SOLDER-COATED COPPLR, GOLD-FLASHED DUMET, OR NICKEL
 806 INCLUDES TYPE BSF
 807 INCLUDES TYPE BSS(S)
 808 INCLUDES TYPE CY(S,H) AND TYPE VY
 809 INCLUDES TYPES CM15, CM20
 810 INCLUDES TYPES DM10 THRU DM30. IND CU LEADS MUST BE SPECIFIED FOR ALL VALUES
 811 RESERVED
 812 INCLUDES TYPES 601PE, 663F, 663FW, 663UW. NONMAG WHEN SUPPLIED W/SPL CU LEADS
 813 INCLUDES TYPE 616G
 814 INCLUDES TYPES LS1 AND AS2
 815 INCLUDES TYPE 182T LEVEL A AND TYPE 182T LEVEL B
 816 LEADS ARE ALLOY 180 WITH GOLD PLATE OVER COPPER PLATE
 817 RESERVED
 818 INCLUDES TYPE 250D
 819 APPLIES TO 125 DEG C SERIES ONLY. NONMAG ONLY W/ALLOY 180 LEADS, SPL ORDER
 820 INCLUDED VALUES TYPES 120D(H), 121D(H), 122D(H), 123D(H), CL33
 821 RESERVED
 822 INCLUDES TYPE VC23G. METAL PARTS PHOSPHOR BRONZE, SILVER, AND INVAR.
 823 KOVAR, DUMET, INVAR, COPPERWELD LEADS ARE EXTREMELY FERROMAGNETIC.
 824 INCLUDES SERIES DI-52, DI-72, DI-42, DI-1515
 825 INCLUDES 1N1583, 1N1585, 1N1587, 1N1124(S), 1N1126(S), 1N1128(S)
 826 INCLUDES 1N2611(S), 1N2613(S), 1N2615(S)-PWR/1N2623A-REF, PREC/1N3016-3030REFGP
 827 INCLUDES 1N645(S,H), 1N647(S), 1N649(S)-PWR/PS52CM(SIG/CMPTK)
 828 INCLUDES SC2, SC4, SC6, SC8, SC10
 829 HI-REL DIODE. SAME PACKAGE AS 1N645
 830 INCLUDES 1N1124, 1N1126, 1N1128
 831 INCLUDES 1N2069, 1N2070, 1N2071
 832 INCLUDES 1N645, 1N647, 1N649
 833 INCLUDES 1N1583, 85, 87-PWR/1N1351, 53, 55, 57, 59-REFGP
 834 INCLUDES 1D118 THROUGH 1D200B
 835 INCLUDES 1N2810A, 1N2820A
 836 INCLUDES PG960B THRU PG971B
 837 COLD-ROLLED STEEL-PACKAGED
 838 INCLUDES 1N1351, 1N1353, 1N1355, 1N1357, 1N1359, 10MZ14ZB1, 10MZ16ZB1
 839 INCLUDES 1N2810A, 1N2820A
 840 INCLUDES 1N3283, 1N756, 1N827A, 1N944B, 1N3286, Z-4C
 841 INCL 1N459A(S-SIG/CMPTK)/1N746A-748A, 750A, 52A, 54A, 56A, 58A(H), 59A(S-REF, GP)
 842 INCL PS4640, 4641, 4653(S,H-REF, GP)/PC107, 112, 115, 116, 126, 137(VARICAPS)
 843 INCLUDES 1N761, 1N3504
 844 INCLUDES 1N1805, 1N1807
 845 INCLUDES 1N2033, 1N2039
 846 INCLUDES 1N3016, 3018, 3020, 3022, 3024, 3026, 3028, 3029, 3030
 847 INCLUDES CD4246, CD4113, SP300100
 848 INCLUDES 1N821/1N822(S)
 849 NO CODE. AMERICAN MICRO DEVICES, INC., PHOENIX 20, ARIZ.
 850 INCLUDES 1N459A(S), 1N750A
 851 INCLUDES FD300, FD643, FD1184(S,H)/FD126, FD1201, FD295, FD306, FD646
 852 INCLUDES HR1B, HF1C177(SIG/CMPTK)/1N1931, 1N1934(REF GP)
 853 INCLUDES MC1291(S,H), MC1739K
 854 INCLUDES TYPES MC-83, 1N3593(TI-2)
 855 SPL 1N3593 W/EPOXY CASE AND PT LEADS
 856 AG ON BRASS/ PHOSPHOR BRONZE

Table III. Remarks (Continued)

857 INCLUDES 8166B, 82088A, 8175
 858 APPLIES TO ALL EXCEPT 8000 SERIES (ALUMINUM)
 859 MUST BE SPECIFIED WITHOUT NICKEL FLASH
 860 MODIFIED TYPE E, MS STYLE
 861 RESERVED
 862 RESERVED
 863 TEST TOTAL QUANTITY OF SPECIAL-ORDER INDUCTORS
 864 HIPERMALLOY CASE
 865 TOROID-WOUND, MOLDED EPOXY CASE
 866 INCLUDES MLF (FLIP-FLOP), MLC (GATE), MLS (HALF-SHIFT REGISTER), MLB (BUFFER)
 867 INCLUDES SN510, SN511 (FLIP-FLOP), SN512 (GATE NAND-NOR), SN515 (GATE OR)
 868 RESERVED
 869 THRU 898 RESERVED
 899 MAGNETIC LATCHING 2PD1
 900 INCLUDES GBT 1/2 AND GBT 1. LEADS ARE ALLOY-COATED COPPER
 901 INCLUDES CEA, CEB, CEC. LEADS ARE ALLOY-COATED COPPER
 902 INCLUDES CG 1/8, CG 1/4.
 903 INCLUDES CDM 1/8, CDM 1/4.
 904 INCLUDES 9855-2, 9852, 9850
 905 INCLUDES EM, MEA (T-O), MEB (T-O), MEC (T-O). LEADS ARE ALLOY-COATED COPPER
 906 INCLUDES RH-10, RH-25, RH-50
 907 INCLUDES RS-1B, RS-2A, RS-2, RS-5, RS-10
 908 INCLUDES M10W, M25W, M50W
 909 INCLUDES TS1W, TS2W, TS3W, TS5W, TS10W
 910 INCLUDES TYPES LAC, LFB, PB
 911 INCLUDES ALL MIL-R-93C TYPES
 912 RESERVED
 913 INCLUDES TYPES EP-20, EP21, 301P, AND ALL OTHER MIL-R-93C
 914 INCLUDES TYPE L-1/2W, TYPE K-3W
 915 INCLUDES ALL HELIPOT TYPES
 916 LEADS ARE GOLD PLATED PRINTED CIRCUIT PINS
 917 RESERVED
 918 AVAIL W/TEFLON WIRE LEADS, AU PLATED SOLDER LUGS, OR AU PLATED CKT PINS
 919 INCLUDES SER 600NM, SER 1500NM
 920 NO CODE. TECHNO-COMPONENTS CORP., NORTHRIDGE, CALIF.
 921 ACTUATED EITHER BY COIL OR PERMANENT MAGNET
 922 RESERVED
 923 INCLUDES FT-SM-8P30, PR300P30, FT-SM-16URP30
 924 INCLUDES DF-101, DF-103
 925 INCL 3600 SER TYPES 3630-2, 4/X3630-2/3631-2, 3/X3631-2/3650-2, 4/X3650-2
 926 INCL 4500SER, 5000SER, TYPES 4505B, D, X4505B, 4535B, C, X4535B, 5075B, D, X5075B
 927 GOLD FLASH, CADMIUM PLATE OVER BRASS
 928 INCL TS-111-6M, TS-112-6M, TS-111-6F, TS-112-6F, TS-111-1, TS-112-2
 929 INCLUDES 622G, 625G
 930 SILVER-PLATED ALUMINUM AVAILABLE
 931 INCLUDES SKT 8-P20, SKT-103PC
 932 INCLUDES TYPE QB41J1, TYPE RA43L1
 933 RESERVED
 934 INCLUDES TYPE 2230, TYPE 5040
 935 INCLUDES SP11, SP21 (DRIVER), SP66 (OUTPUT-ISOLATION)
 936 INCLUDES DO-TS10, DO-T19, DO-T10, DO-T8
 937 INCLUDES TYPES NS-3001, 3N70
 938 INCLUDES TYPES 2N941 THOUGH 2N946 (CHOPPER), 2N940 (SM SIG)
 939 INCLUDES SM2704, 2N2432
 940 INCLUDES 2N2060, S4371, S4372
 941 AVAILABLE IN TO-51 GLASS PACKAGE WITH PLATINUM RIBBON LEADS

Table III. Remarks (Continued)

942 INCL. 2N708(S,H), 718A, 722(H), 869, 910(S,H), 911(S), 912(S), 914, 915(S), 916, 956
943 INCL. 2N995, 2369, 2484/S4371(H), S4372(H)
944 INCL. 2N1131(S), 1132(H), 1613(S,H), 1711(S), 1890, 1893(S,H), 1973, 2049, 2297(S)
945 INCLUDES S4286(H), S4374
946 INCLUDES 2N2222(S,H), 2N2501
947 INCL. 2N1613(S,H), 1711(S), 1893(S,H) 657, 1132-MED PWR/2N1506-SM SIG
948 INCLUDES 2N2497, 2498-FET
949 INCL. 2N1016A(S), 1016B, 1016C, 1016D
950 INCLUDES 2N2035, 2N2036
951 INCLUDES 2N389(S), 2N424
952 INCLUDES 2N1048B, 2N1050B, SP926
953 SPECIAL 2N1714 IN 2N1718 DOUBLE-ENDED STUDMOUNT PACKAGE
954 INCLUDES 2N389(S), 2N424
955 INCLUDES 2N1016A(S), 1016B, 1016C, 1016D, 2226
956 NONMAGNETIC VERSION OF 2N1506A
957 INCLUDES 2N1234, 2N1257
958 INCLUDES 2N328A, 2N329A (S)
959 INCLUDES 2N859, 861, 865, 2185, 2278
960 INCL. 2N328A, 2N329A(S)/2N1026, 2N1469, 2N1917
961 INCL. 2N930, 2412(S,H)/718A, 780, 740-SM SIG/2N956-MED PWR
962 INCLUDES 2N338, 2N2192A, 2N2323M
963 INCLUDES 2N886 2N897
964 INCLUDES 3C60A(TRIGISTOR)/3A101, 3A201A(CONTROLLED SWITCH)
965 INCLUDES 2N491B, 2N492B, MM/2N491B
966 INCLUDES 50-3946, 50-3947
967 INCLUDES RG-55B/U, RG-58C/U, RG-210/U, RG-108A/U
968 NO CODE. VICTOR ELECT. WIRE + CABLE CORP., W. WARWICK, R.I.
969 LEADS: 5EA, RG-8/U
97 NO CODE. W.L. GORE AND ASSOCS., INC., NEWARK, DELAWARE
971 INCLUDES 1XT-20-1932(2)SFJ, 1XT-20-728STJ
972 SINGLE AND DUAL CONDUCTOR, INS: TEFLON, COND: AU/CU
973 CONDUCTOR: TND CU
974 NO CODE. INDIANA GENERAL CORP., MAGNET DIV., VALPARAISO, IND.

Table IV. Federal Stock Codes

00614 LEACH CORP., COMPTON, CALIFORNIA
 00656 AEROVOX CORP., NEW BEDFORD, MASS.
 00872 AMELCO, INC., MOUNTAIN VIEW, CALIFORNIA
 01121 ALLEN BRADLEY CO., MILWAUKEE 4, WISCONSIN
 01281 PACIFIC SEMICONDUCTOR, INC., CULVER CITY, CALIFORNIA
 01295 TEXAS INSTRUMENTS, INC., SEMICONDUCTOR-COMP. DIV., DALLAS, TEXAS
 01961 PULSE ENGINEERING, INC., SANTA CLARA, CALIFORNIA
 02111 SPECTROL ELECTRONICS CORP., SAN GABRIEL, CALIFORNIA
 02735 RADIO CORP. OF AMERICA, SEMICOND. AND MATERIALS DIV., SOMERVILLE, N.J.
 02985 TEPKO ELECTRIC CORP., ROCHESTER 4, NEW YORK
 03034 PENN-KEYSTONE CORP., DERBY, CONN.
 03890 MARKEL, L. FRANK AND SONS, NORPISTON, PA.
 03911 CLAIREX CORP., NEW YORK 1, N.Y.
 04099 CAPCO CAPACITORS, DIV. TEXTOL PRODUCTS, IRVING, TEXAS
 04426 LICON DIV., ILLINOIS TOOL WORKS, CHICAGO 34, ILL.
 04454 LITTON INDUSTRIES, INC., COMPONENTS DIV., MOUNT VERNON, NEW YORK
 04713 MOTOROLA SEMICONDUCTOR PRODUCTS, INC., PHOENIX, ARIZ.
 04867 HIRAM JONES ELECTRONICS, BURBANK, CALIFORNIA
 05079 TRANSITOR ELECTRONICS, INC., BENNINGTON, VT.
 05277 WESTINGHOUSE ELECTRIC CORP., SEMICOND. DIV., YOUNGWOOD, PA.
 05397 KEMET DIV., UNION CARBIDE AND CARBON CORP., CLEVELAND 1, OHIO
 05791 LYN-TRON, INC., NORTH HOLLYWOOD, CALIFORNIA
 05973 AMERICAN SUPER-TEMPERATURE WIRES, INC., WINDOOSKI, VT.
 06090 RAYCHEM CORP., REDWOOD CITY, CALIFORNIA
 07088 KELVIN ELECTRIC, INC., VAN NUYS, CALIFORNIA
 07145 TIMES WIRE AND CABLE DIV., INTERNATIONAL SILVER CO., WALLINGFORD, CONN.
 07150 G.B. COMPONENTS, INC., VAN NUYS, CALIFORNIA
 07180 SAGE LABS. INC., NATICK, MASS.
 07256 SILICON TRANSISTOR CORP., CARLE PLACE, N.Y.
 07263 FAIRCHILD SEMICONDUCTOR, MOUNTAIN VIEW, CALIFORNIA
 07497 FXR DIV., AMPHENOL-BORG ELECTRONICS CORP., DANBURY, CONN.
 07713 SPERRY SEMICONDUCTOR DIV. SPERRY RAND CORP., NORWALK, CONN.
 07716 INTERNATIONAL RESISTANCE CORP., BURLINGTON DIV., BURLINGTON, IOWA
 07910 CONTINENTAL DEVICE CORP., HAWTHORNE, CALIFORNIA
 08145 U.S. ENGINEERING CO., DIV. LITTON INDUSTRIES, INC., VAN NUYS, CALIF.
 08732 SOLID STATE PRODUCTS, INC., SALEM, MASS.
 08795 RAYCLAD TUBES, INC., REDWOOD CITY, CALIFORNIA
 08798 GENERAL ELECTRIC CO., IND. SALES OPN. SECT 998-75, SCHENECTADY, NEW YORK
 08806 GENERAL ELECTRIC CO., MINIATURE LAMP DEPT., CLEVELAND 12, OHIO
 09026 BABCOCK RELAY DIV., BABCOCK ELEC. CORP., COSTA MESA, CALIFORNIA
 09132 DAYSTROM, INC., CONTROL SYSTEMS DIV., INC., LA JOLLA, CALIFORNIA
 09213 GENERAL ELECTRIC CO., SEMICONDUCTOR PRODUCTS DEPT., SYRACUSE, N.Y.
 09214 GENERAL ELECTRIC CO., RECTIFIER COMPONENTS DEPT., AUBURN, NEW YORK
 09349 MAGNETIC CIRCUIT ELEMENTS, INC., MONTROSE, CALIFORNIA
 11313 REED AND REESE, INC., PASADENA, CALIFORNIA
 11530 PHILCO CORP., WESTERN DEVELOPMENT LABS, PALO ALTO, CALIFORNIA
 12040 NATIONAL SEMICONDUCTOR CORP., DANBURY, CONN.
 12060 DIODES, INC., CANOGA PARK, CALIFORNIA
 12065 TRANSITRON ELECTRONIC CORP., WAKEFIELD, MASS.
 12401 INTERNATIONAL RESISTANCE CORP., PHILADELPHIA DIV., PHILADELPHIA, PENN.
 12515 THERMATICS, INC., ELM CITY, N.C.
 12617 HAMLIN, INC., LAKE MILLS, WISCONSIN
 13088 TERMINAL DESIGNS, INC., N. ARLINGTON, N.J.
 13934 MIDWEC CORP., OSHKOSH, NEBRASKA
 14099 SEMTECH DIV. OF CONTINENTAL DEVICE, NEWBURY PARK, CALIFORNIA
 14552 MICRO SEMICONDUCTOR CORP., CULVER CITY, CALIFORNIA

Table IV. Federal Stock Codes (Continued)

14604 ELMWOOD SENSORS, INC., CRANSTON 7, R.I.
 14674 CORNING GLASS, CORNING, N.Y.
 15116 MICRODOT, INC., SOUTH PASADENA, CALIFORNIA
 15287 SCIONICS CORP., CANOGA PARK, CALIFORNIA
 15450 ERIE ELECTRONICS DIV., ERIE RESISTOR CORP., ERIE, PENN.
 15801 FENWAL ELECTRONICS, INC., FRAMINGHAM, MASS.
 18626 DRIVER-HARRIS CO., HARRISON, N.J.
 21520 FANSTEEL METALLURGICAL CORP., N. CHICAGO, ILL.
 45402 PACIFIC SCIENTIFIC CO., LOS ANGELES 22, CALIFORNIA
 56289 SPRAGUE ELECTRIC, NORTH ADAMS, MASS.
 63060 VICTOREEN INSTRUMENT CO., CLEVELAND 3, OHIO
 71400 BUSSMAN MFG. DIV., MCGRAW-EDISON CO., ST. LOUIS 7, MO.
 71468 CANNON ELECTRIC CO., LOS ANGELES 31, CALIFORNIA
 71482 C.P. CLARE AND CO., HOLLYWOOD, CALIFORNIA
 71590 CENTRALAB DIV. OF GLOBE UNION, INC., MILWAUKEE, WISCONSIN
 71785 CINCH MFG CORP., CHICAGO, ILL.
 72259 NYTRONICS, INC., ESSEX ELECTRONICS DIV., BERKELEY HEIGHTS, NEW JERSEY
 72354 J. E. FAST AND CO., CHICAGO, ILL.
 72699 GENERAL INSTRUMENT, SEMICONDUCTOR DIV., ELIZABETH, N.J.
 72928 GUDAMAN, INC., CHICAGO, ILL.
 73168 FENWAL, INC., ASHLAND, MASS.
 73293 HUGHES AIRCRAFT CO., SEMICONDUCTOR DIV., NEWPORT BEACH, CALIFORNIA
 73899 JFD ELECTRONIC CORP., COMPONENTS DIV., BROOKLYN, N.Y.
 74970 E.F. JOHNSON CO., WASECA, MINN.
 76433 MICAMOLD RADIO, BROOKLYN, NEW YORK
 77342 POTTER AND BRUMFIELD, DIV. OF A.M.F. CO., PRINCETON, IND.
 77764 RESISTANCE PRODUCTS CO., HARRISBURG, PENN.
 77820 BENDIX CORP., SCINTILLA DIV., SIDNEY, NEW YORK
 78277 SIGMA INSTRUMENTS, INC., BRAINTREE, MASS.
 80223 UNITED TRANSFORMER CORP., NEW YORK 13, NEW YORK
 80294 BOURNS, INC., TRIMPOT DIV., RIVERSIDE, CALIFORNIA
 80740 BECKMAN INSTRUMENTS, INC., HELIPOT DIV., FULLERTON, CALIFORNIA
 81095 TRIAD TRANSFORMER CORP., DIV. LITTON INDUSTRIES, INC., VENICE, CALIFORNIA
 81453 RAYTHEON CO., INDUSTRIAL COMPONENTS DIV., NEWTON 58, MASS.
 82110 GUDEBROD BROS. SILK CO. INC., ELECTRONICS DIV., PHILADELPHIA 7, PENN.
 82647 TEXAS INSTRUMENTS, INC., METALS AND CONTROLS DIV., ATTLEBORO, MASS.
 82879 ROYAL ELECTRIC CORP., PAWTUCKET, R.I.
 83186 VICTORY ENGINEERING CO., SPRINGFIELD, N.J.
 84171 ARCO ELECTRONICS CO., GREAT NECK, NEW YORK
 88997 UNION SWITCH AND SIGNAL DIV., WESTINGHOUSE AIR BRAKE CO., PITTSBURGH, PA.
 89037 GOOD-ALL DIV. TRW ELECTRONICS, CHICAGO, ILL.
 90484 SURPRENANT MFG. CO., CLINTON, MASS.
 91418 RADIO MATERIAL CORP., CHICAGO, ILL.
 91637 DALE ELECTRONICS, INC., COLUMBUS, NEBR.
 91662 ELCO, CORP., WILLOW GROVE, PENN.
 91929 HONEYWELL, MICRO SWITCH DIV., FREEPORT, ILL.
 91984 MAIDA DEVELOPMENT CO., PHOEBUS, VA.
 93332 SYLVANIA ELECTRIC PRODUCTS, INC., SEMICONDUCTOR DIV., WOBURN, MASS.
 93410 STEVENS MFG. CO., INC., MANSFIELD, OHIO
 93738 TELERADIO ENGINEERING CORP., WILKES BARRE, PENN.
 93929 G-V CONTROLS, INC., LIVINGSTON, N.J.
 94145 RAYTHEON CO., SEMICONDUCTOR DIV., MOUNTAIN VIEW, CALIFORNIA
 94875 ELECTRO TEC CORP., SOUTH HACKENSACK, NEW JERSEY
 95077 GENERAL RF FITTINGS CO., BOSTON, MASS.
 95275 VITRAMON, INC., BRIDGEPORT, CONN.
 95712 DAGE ELECTRIC CO. INC., FRANKLIN, IND.

Table IV. Federal Stock Codes (Continued)

96341 MICROWAVE ASSOCIATES, INC., BURLINGTON, MASS.
96733 WESTCAP DIV., SAN FERNANDO ELECTRIC MFG. CO., SAN FERNANDO, CALIFORNIA
97979 REON RESISTOR CORP., YUNKERS, NEW YORK
98053 WARREN WIRE CO., PQWNAL, VT.
98291 SEAELECTRO CORP., MAMARONECK, N.Y.
98659 COMPUTER INSTRUMENTS CORP. HEMPSTEAD, L.I., NEW YORK
99114 HI-TEMP WIRES, INC., WESTBURY, N.Y.
99120 PLASTIC CAPACITORS, INC., CHICAGO, ILL.
99127 BALCO CAPACITOR DIV., BALCO RESEARCH LABS, NEWARK, N.J.
99217 SOUTHERN ELECTRONICS CO., BURBANK, CALIFORNIA
99515 ELECTRON PRODUCTS, LOS ANGELES, CALIFORNIA
99699 FILTERS, INC., NORTHPORT, NEW YORK
99800 DELEVAN ELECTRONICS CORP., EAST AURORA, NEW YORK
99942 HOFFMANN ELECTRONICS CORP., SEMICONDUCTOR DIV., EL MONTE, CALIFORNIA

An "X" in any column indicates that a remark card in column 13 applies to the column containing the "X."

Abbreviations used on the parts lists and remarks cards are given in the pages immediately following this introduction. Wherever possible, abbreviations used were obtained from MIL-STD-12B, Abbreviations for Use on Drawings and in Technical-Type Publications.

C. Abbreviations Used on Parts List IBM Cards

1. Component Class Abbreviations

Capacitor	CAP
Connector	CONN
Controlled Rectifier	SCR
Core	CORE
Crystal	XTAL
Diode	DIODE
Fuse	FUSE
Hardware	HDW
Inductor	IND
Lacing Cord	LCNG CD
Lamp	LAMP
Microcircuit	UCT
Motor	MOTOR
Relay	RELAY
Resistor, Fixed	RES FXD
Resistor, Variable	RES VAR
Switch	SWITCH
Terminal	TERM
Thermistor	TMTR
Thermostat	THERMO
Transformer	XFMR
Transistor	TSTR
Tube, Electron	TUBE EL
Wire and Cable	W & CBL

2. Component Type Abbreviations

Blocking Oscillator	BLKG OSC
Carbon Composition	C COMP
Carbon Film	C FLM
Ceramic, low voltage	CER LV
Choke, RF	CHOKE RF
Chopper	CHOPPER
Class 1, 2, 3	CL 1, 2, 3
Coaxial	COAX
Dual Transistor	DUAL TSTR
Electrometer	ELECTROM
Field Effect Transistor	FET
Glass or Porcelain	GL/PORC
General Purpose Carbon Composition	GP C COMP
Hookup	HKUP
Hookup, Shielded	HKUP SHLD
High-Temperature	HI TEMP
Interstage	INTRSTG
Limit, Precision	LIM PREC
Low Current	LO CUR
Low Frequency	LF
Medium Power	MED PWR
Metal Film	MET FLM
Metallized Mylar	MET MYLAR
Metallized Paper	MET P
Mica	MICA
Micrologic Series	ULOGIC SER
Microminiature, Painted	UMIN, PNTD
Microwave Mixer	UWAVE MXR
Microwave Varactor	UWAVE VRCR
Mylar	MYLAR
Non-Magnetic	NONMAG

Paper	P
Paper, Feedthrough	P FTRU
Paper, Military	P MIL
Paper, MIL, Characteristic K	P MIL(K)
Plastic	PLAS
Power	PWR
Power, Bridge	PWR BRDG
Precision, Carbon Film	PREC C FLM
Precision, Wirewound	PREC WW
Printed Circuit Board	PCB
Quick-Disconnect	Q DISC
Reference, General Purpose	REF GP
Reference, Precision	REF PREC
Round, Multipin	RD MLTIPIN
Sensitive	SENSTV
Shielded Twisted Pair	SHLD TWISTED PR
Signal and Computer	SIG/CMPTR
Small Signal	SM SIG
Snap Action	SNP ACTN
Style 18-Dacron 96	ST 18-DAC96
Tantalum, Dry	TA DRY
Tantalum, Wet Foil	TA WT FOIL
Tantalum, Wet Slug	TA WT SLUG
Trimmer, Air Variable	TRM AIR VAR
Trimming Carbon, Composition	TRM C COMP
Trimming, Wirewound	TRM WW
Tunnel	TUNNEL
Unijunction Transistor	UJT
Very High Frequency	VHF

Wirewound, Precision, WW PREC ENC
Encapsulated

Wirewound, Power, WW PWR PREC
Precision

3. Package Abbreviations

Bell, Miniature	BELL MIN
Ceramic, Miniature	CER MIN
Crystal can	XTAL CAN
Diamond, Special	DMND SPL
Disc, Standard	DISC STD
Epoxy, coated	EPXY CTD
Epoxy, molded	EPXY MLD
Feedthrough, standard	FTRU STD
Flangeless	FLNGLESS
Glass, subminiature	GL SMIN
Grounding, Studmount	GRDSTDMT
Hermetically sealed	HERM SLD
Microdiode	UDIODE
Microminiature	UMIN
Plastic, molded	PLAS MLD
Rectangular, miniature	RECT MIN
Rectangular, subminiature	RECT SMIN
Rectangular, Standard	RECT STD
Round	RD
Special	SPL
Special, glass	SPL GL
Standoff, standard	STOFFSTD
Studmount	STDMT
Tubular, adjustable	TUB, ADJ
Tubular, feedthrough	TUB FTRU
Tubular, flattened	TUB FLT

Tubular, miniature	TUB MIN
Tubular, subminiature	TUB SMIN

4. Lead Material and Associated Abbreviations

Aluminum	AL
Anodized aluminum	ANDZD AL
Beryllium copper	BE CU
Brass	BRS
Copper	CU
Copperweld	CUWELD
Enameled wire	ENAM W
Epoxy	EPXY
Gold	AU
Insulation	INS
Iron	FE
Irradiated polyolefin	IPO
Jacket	JKT
Nickel	NI
Phosphor bronze	PH BRZ
Plastic	PLAS
Platinum-iridium	PT/IR
Polyethylene	POLYETH
Rhodium	RH
Silver	AG
Stainless steel	STLS STL
Tantalum	TA
Tinned copper	TND CU
Tinned dumet	T/DUMET

APPENDIX B
EXPLANATION OF STATISTICAL
TERMS

APPENDIX B

EXPLANATION OF STATISTICAL TERMS

Often in analytical problems, functional relationships relate several variables; yet, specific values for these variables may only be determined statistically. Such variables are referred to as random or stochastic variables.

In order to characterize random variables, statistical properties must be available. A basic statistical concept is the probability density function. Consider a random variable which takes on various values of y between $-\infty$ and $+\infty$ at consecutive instants. At any one instant, it is absolutely certain that y will take on some value within this range. If quantum levels of width Δy are established along the y -axis and the number of occurrences in each quantum level is tabulated for a total of N occurrences, then the result is the familiar statisticians histogram. Figure B-1 shows how one might form a histogram from a sampled function.⁴ In Figure B-1a beads on a wire represent sampled values of $y(n)$ for different instants; each wire represents a quantum level. The bead rack is then tipped in Figure B-1b, forming a histogram, Figure B-1c. As the width, Δy , between wires becomes infinitely small, the number of samples, N , becomes infinitely large, and if the function $y(n)$ depicts a random variable, then the histogram becomes a probability density function of $y(n)$ which may be designated as $p(y)$.

Probability density functions are normalized so that they have unit area; that is

$$\int_{-\infty}^{+\infty} p(y) dy = 1. \quad (69)$$

The infinitesimal area $p(y) dy$ is that fraction of values of $y(n)$ which are in the interval y to $y + dy$. This area is referred to as the probability of y occurring between some y and $y + dy$. The probability of y occurring between the values a and b , which is often expressed as $P(a \leq y \leq b)$, may then be written as

$$P(a \leq y \leq b) = \int_a^b p(y) dy. \quad (70)$$

Then, $P(-\infty \leq y \leq \infty) = 1$ (is a certainty) from Equation (69). Since the measure of probability is a positive number from 0 to 1, the function $p(y)$ cannot have negative values.

The probability density function of a random variable is especially important because most all of the other statistical parameters may be derived from it.

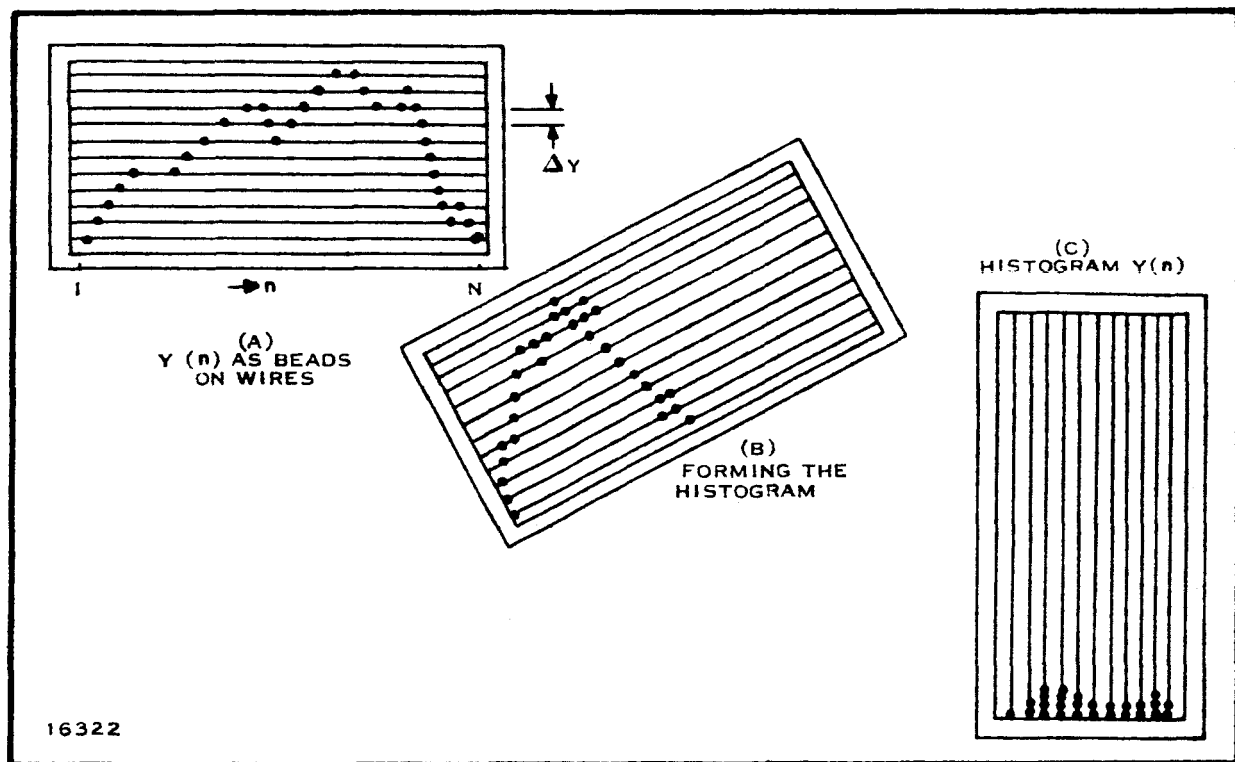


Figure B-1. Formation of a Histogram

One useful class of parameters may be obtained by taking the scalar product of $p(y)$ with $g(y)$, where $g(y)$ is a function of the random variable y . This operation may be expressed by the relation

$$\langle g(y), p(y) \rangle = \int_{-\infty}^{\infty} g(y) p(y) dy . \quad (71)$$

Where $g(y) = y^n$, Equation (71) yields the n^{th} moment of the random variable. The most familiar moment is the first moment ($n = 1$) which is also known as the mean or average value of the random variable. Where M_1 is designated as the first moment,

$$M_1 = \int_{-\infty}^{\infty} y p(y) dy . \quad (72)$$

Other useful parameters which may be obtained from Equation (71) are the central moments. The n^{th} central moment is obtained when $g(y) = (y - M_1)^n$. The most familiar central moment is obtained when $n = 2$, yielding the variance, V , of the probability distribution

$$V = \int_{-\infty}^{\infty} (y - M_1)^2 p(y) dy . \quad (73)$$

The square root of the variance is known as the standard deviation; to electrical engineers it is known as the RMS value of the random variable.

The characteristic function, $C(u)$, of the random variable is obtained by setting $g(y)$ equal to the complex function e^{iyu} .

$$C(u) = \int_{-\infty}^{\infty} e^{iyu} p(y) dy . \quad (74)$$

This operation will be recognized as a Fourier transform. The characteristic function is useful for specifying the number of quantization levels required for sampling data.

Another function useful in statistics is the probability distribution function of y which may be designated as $P(Y)$ and defined by

$$P(Y) = P(-\infty \leq y \leq Y) = \int_{-\infty}^Y p(y) dy . \quad (75)$$

Conversely, the probability density function may be defined as the derivative of the probability distribution function.

$$p(y) = \frac{d}{dy} P(y \leq Y) . \quad (76)$$

The concept of probability density functions and probability distributions may also be extended to multiple random variables. Consider the two-dimensional space of x and y . There exists some probability that a point in the x, y plane having coordinants x and y lies in the region where $x \leq X$ and $y \leq Y$. This probability may be expressed as $P(x \leq X, y \leq Y)$, a joint probability distribution function. A joint probability density function may then be defined as

$$p(x, y) = \frac{\partial^2}{\partial X \partial Y} P(x \leq X, y \leq Y) . \quad (77)$$

It follows that

$$P(x_1 \leq x \leq x_2, y_1 \leq y \leq y_2) = \int_{x=x_1}^{x_2} \int_{y=y_1}^{y_2} p(x, y) dy dx . \quad (78)$$

Also,

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(x, y) dy dx = 1 . \quad (79)$$

A joint probability density function may be reduced to a probability density function of one variable by integrating over the entire range of the variable to be removed.

$$p(y) = \int_{x=-\infty}^{\infty} p(x, y) dx . \quad (80)$$

Joint probability density functions may be extended to the case of k -dimensional random variables.

A problem often encountered (such as the subject of this report) is, given a functional relationship between two variables, such as $y = f(x)$ where x is a real random variable defined by a probability density function, determining the probability density function of y .

Consider a function of which maps each point x of sample space A into one and only one point y of sample space B . Then each set of points in A , $x(S)$, corresponds to a set of points in B , $y(S)$. The probability of obtaining each point of $y(S)$ is equal to obtaining the corresponding point of $x(S)$. Where $y(S)$ is a set over all of region B forming the line segment which is the range of the random variable y , the probability of obtaining a sample point from region B is the integral of the probability density function of $y(S)$ defined over the range of B and also equal to the integral of the probability density function of $x(S)$ defined over the corresponding range of A . Then,

$$P(y \in B) = \int_B p[y(S)] dy = \int_A p[x(S)] dx \quad (81)$$

or

$$\int_B p(y) dy = \int_A p(x) dx . \quad (82)$$

Where x can be expressed as a function of y , such as $x = g(y)$, then $p(y)$ may be expressed in terms of $p(x)$ by a change of variable.⁵

$$\int_A p(x) dx = \int_B p[x = g(y)] |J| dy \quad (83)$$

where the Jacobian, J , is defined by

$$J = \frac{\partial f(x)}{\partial y} \quad (84)$$

Then, from Equations (82), (83), and (84),

$$p(y) = p[x = g(y)] \left| \frac{\partial f(x)}{\partial y} \right| \quad (85)$$

This technique may be extended to functions of multiple random variables.

The probability density function, $p(y)$, may also be obtained by using Equation (76). In the case where $y = f(x)$ is a monotonic increasing function as x increases positively,

$$p(y) = \frac{d}{dy} P(-\infty \leq y \leq Y) = \frac{d}{dy} P\{-\infty \leq x \leq [Y = f(X)]\} \quad (86)$$

or

$$p(y) = \frac{d}{dy} \int_{-\infty}^{Y=f(X)} p(x) dx \quad (87)$$

This technique may also be extended to functions of multiple random variables.

When a new random variable is equal to the sum of two independent random variables, as $y = x_1 + x_2$, the probability density function of y is given by the convolution of the probability density functions of x_1 and x_2 . This may be shown by extending Equation (17) to include the two-dimensional case. Let y and u be defined as functions of x_1 and x_2 .

$$\left. \begin{aligned} y &= x_1 + x_2 \\ u &= x_1 \end{aligned} \right\} \quad (88)$$

Then,

$$\left. \begin{aligned} x_2 &= y - u \\ x_1 &= u \end{aligned} \right\} \quad (89)$$

Equation (85) becomes

$$p(y, u) = p(x_2 = y - u, x_1 = u) \left| \frac{\partial(x_2, x_1)}{\partial(y, u)} \right| \quad (90)$$

$$\frac{\partial(x_2, x_1)}{\partial(y, u)} = \left| \begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix} \right| = 1; \quad (91)$$

so

$$p(y, u) = p(x_2 = y - u, x_1 = u). \quad (92)$$

Since x_1 and x_2 are independent,

$$p(x_1, x_2) = p(x_1) p(x_2) \quad (93)$$

by the theorem of compound probability. Then,

$$p(y, u) = p(y - u) p(u). \quad (94)$$

By Equation (80),

$$p(y) = \int_{u=-\infty}^{\infty} p(y - u) p(u) du. \quad (95)$$

This integral is known as the convolution integral.